TECHNOLOGY DEPT.

THE ROYAL AERONAUTICAL SOCIETY WAS FOUNDED IN 1866

JANUARY, 1943
No. 385
Vol. XLVII

MONTHLY
PRICE
5/-

PUBLIC LIBRARY
APR 9 - 1943
DETROIT

The Journal of The Royal Aeronautical Society

With which is incorporated The Institution of Aeronautical Engineers

Edited for the Council by Capt. J. LAURENCE PRITCHARD, Hon. Fellow

CONTENTS FOR JANUARY, 1943

Monthly Notices

Dynamic Stiffness and Effective Inertia Methods in Modern Torsional Vibration Theory - *Manley*

The Electro-Mechanical Analogy in Oscillation Theory - - - - - - - - *Manley*

Reviews

Abstracts from the Scientific and Technical Press

Titles and References of Articles and Papers

The ROYAL AERONAUTICAL SOCIETY, 4, Hamilton Place, W.1

Telephones: Grosvenor 3515 (3 lines)

AERONAUTICAL BOOKS

BENNETT'S COMPLETE AIR NAVIGATOR

By D. C. T. BENNETT, D.S.O. This is a fourth edition of a standard work. It deals fully with astronomical navigation, air pilotage and direction-finding wireless, together with a certain amount of calculation. It includes many other subjects of importance to the navigator and a most valuable chapter on Meteorology. There are over 140 diagrams, some in colour. Fourth Edition.

15s. net.

ESSENTIAL METALLURGY FOR ENGINEERS

By A. C. VIVIAN, B.A. (Cantab.), etc. Lecturer in Metallurgy at the R.A.F. School of Aeronautical Engineering. In this book the reader is given a short account of the essentials in modern control over the properties of metallic materials. It is a book, devised on new lines, aiming at the correct, complete but easy understanding by technicians of the diverse metallic alloys of engineering. Simplifications are its strong features, including a unification of all "heat treatment" into one system, which safely brings this complex matter within the reach of all.

8s. 6d. net.

D/F HANDBOOK FOR WIRELESS OPERATORS

By W. E. CROOK. This book is written in an easily readable style that will impress every detail on the minds of readers interested in this subject. The book will, in Crook's own words, "enable the reader to view any D/F station through the specialist's eye and to take up his duties with the minimum of delay necessary to master local details."

3s. 6d. net.

PITMAN'S, PARKER ST., KINGSWAY, W.C.2

TREFOIL

BAKELITE PLASTICS

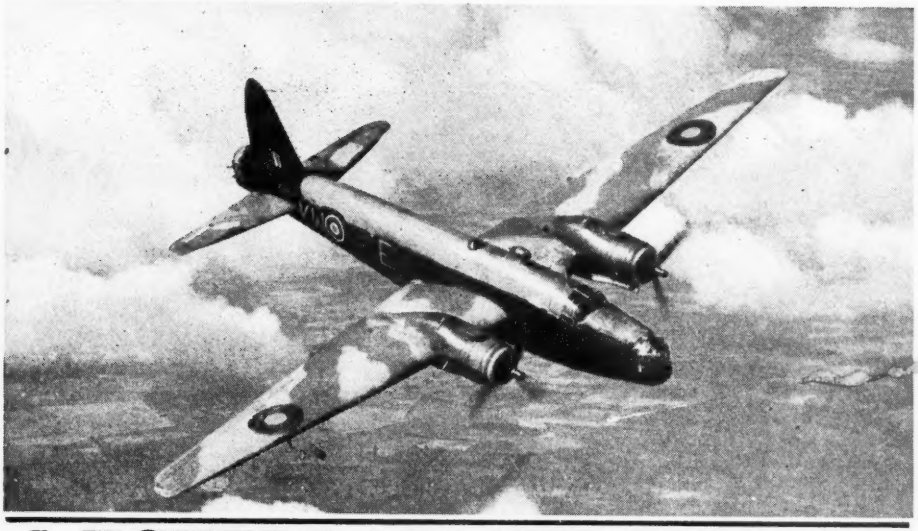
REGD. TRADE MARKS

Pioneers in the Plastics World

STRIP STEEL

SPECIALISTS IN THE ROLLING AND
—— HEAT TREATMENT OF ——
AIRCRAFT STRIP AND SHEET STEEL

J. J. HABERSHON & SONS, Ltd.
HOLMES MILLS, ROTHERHAM



VICKERS-ARMSTRONGS LTD
Head Office: VICKERS HOUSE · BROADWAY · LONDON · S·W·1

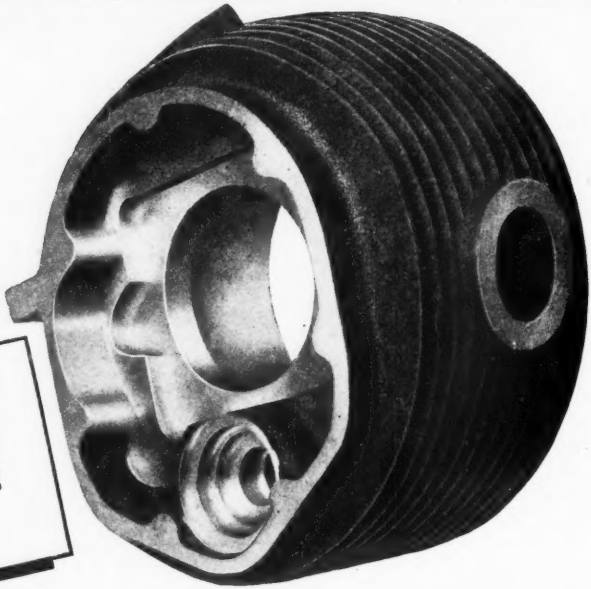
"Aeroplane" photograph

DUNLOP
AEROPLANE
TYRES
WHEELS
& BRAKES

2H/609

CYLINDER HEAD
IN
"ELEKTRON"
MAGNESIUM
ALLOY

By Courtesy of
Messrs. F. M. Aspin & Co.

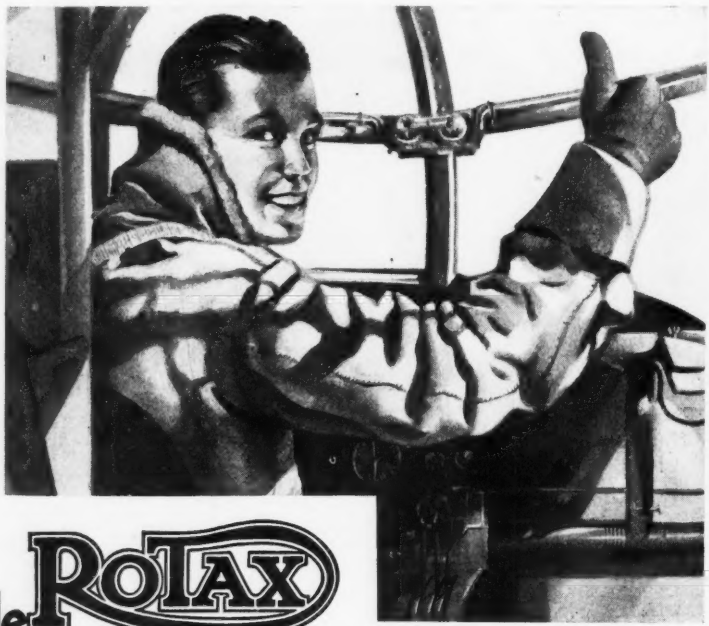


ELEKTRON
THE PIONEER MAGNESIUM ALLOYS

● *Sole Producers and Proprietors of the Trade Mark "Elektron":* MAGNESIUM ELEKTRON LIMITED, Abbey House, London, N.W.1 ● *Licensed Manufacturers: Castings & Forgings:* STERLING METALS LIMITED, Northey Road, Foleshill, Coventry ● *Castings:* THE BIRMINGHAM ALUMINIUM CASTING (1903) COMPANY LIMITED, Birmid Works, Smethwick, Birmingham ● J. STONE & COMPANY LIMITED, Deptford, London, S.E.14 ● *Sheet, Extrusions, Forgings & Tubes:* JAMES BOOTH & CO. (1915) LIMITED, Argyle Street Works, Nechells, Birmingham, 7 ● *Sheet, Extrusions, Etc.:* BIRMETALS LIMITED, Woodgate, Quinton, Birmingham ● *Suppliers of Magnesium and "Elektron" Metal for the British Empire:* F. A. HUGHES & CO. LIMITED, Abbey House, Baker Street, London, N.W.1

for Aircraft

MAGNETOS,
STARTERS,
GENERATORS,
LANDING LAMPS,
NAVIGATION
LAMPS,
INTERIOR
LIGHTING,
BATTERIES,
SWITCHGEAR,
DE-ICING AND
ANTI-ICING
EQUIPMENT
BOOSTER COILS,
SMALL MOTORS,
VACUUM PUMPS,
RADIO
GENERATORS
ETC.



Dependable ROTAX
ELECTRICAL EQUIPMENT
ROTAX LTD. · WILLESDEN JUNCTION · LONDON, N.W.10

The Heart of the Aeroplane

The heart of the aeroplane is its engine . . . Aero engine development is as vital as that of new air-frames . . . Shining brightest of all is the Rolls-Royce Merlin which saved the world in the Battle of Britain two years ago and today, in improved form, is being built both in Great Britain and the U.S.A.

Sunday Times, Sept. 20th.

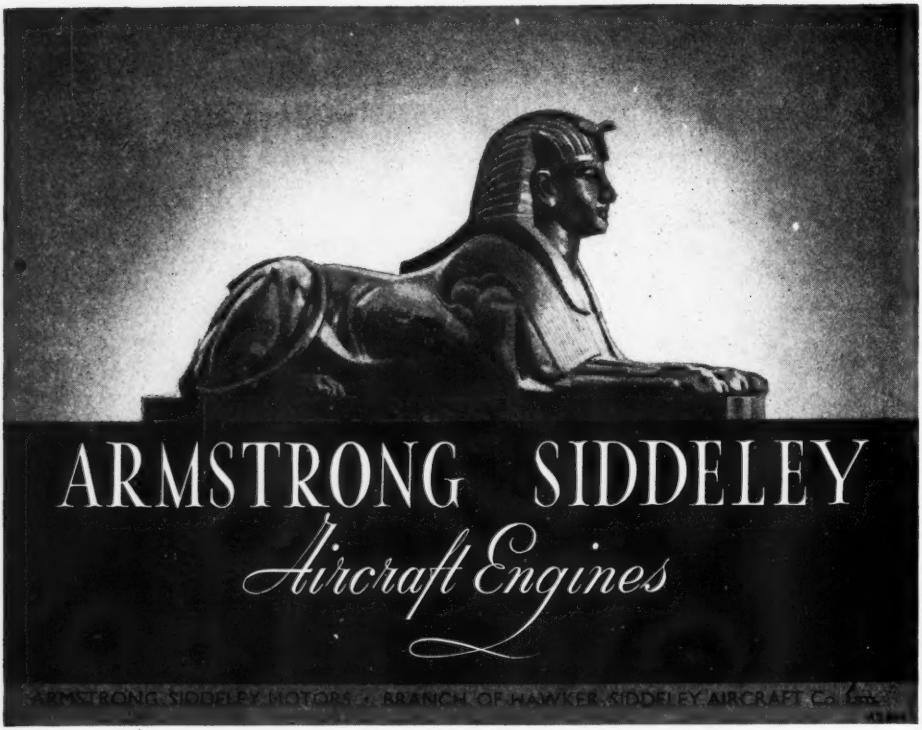
ROLLS-ROYCE

AERO ENGINES

FOR SPEED AND RELIABILITY

CUNLIFFE-OWEN AIRCRAFT LIMITED

AIRCRAFT DESIGNERS & CONSTRUCTORS, ENGLAND



B T-H

ELECTRICAL EQUIPMENT
for AIRCRAFT

The BTH Company manufactures all kinds of electrical equipment for Aircraft including :—

Starting Equipment ; Generators ; Air Compressors ; Engine Speed-indicators ; Undercarriage and Flap Operating Equipment ; Petrol Pump Motors ; Mazda Aircraft Lamps for Landing, Navigation, and Interior Lights.

• • •

BTH Electrical Equipment is fitted to many well-known types of aircraft

BTH THE BRITISH THOMSON-HOUSTON CO., LTD.
CROWN HOUSE, ALDWYCH, LONDON, W.C.2.

A2922N

PRECISION

in design, manufacture and testing: the basis of Lockheed success.

AUTOMOTIVE PRODUCTS COMPANY LIMITED

LOCKHEED

Trade Mark

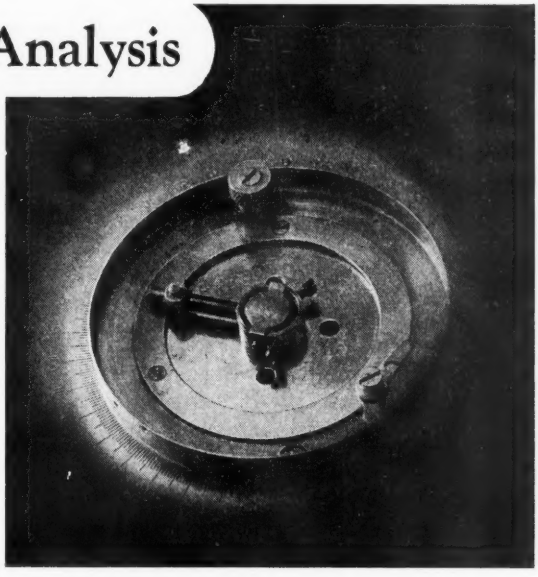
HYDRAULIC UNITS

Crystal Analysis

CRYSTAL analysis is the scientific approach to the atomic structure of substances. The apparatus employed is an X-ray spectrographic camera which enables photographs of X-ray diffraction by crystals of magnesium to be made. The axis of the arm shown in the illustration continues down into the centre of a cylindrical chamber and on it is mounted the crystalline specimen. This is bathed in a monochromatic X-ray beam and a photograph of the diffraction pattern made by the reflected rays is taken on a film which lines the chamber wall.

The application of the science of crystallography to the metallurgy of magnesium and its alloys has led to our present knowledge of the structure of the magnesium crystal, the number of phases which exist in the alloys, the orientation of crystals in worked material and measurement of internal stresses.

The results of this research are constantly applied in practice to maintain for MAGNUMINIUM magnesium alloys their position in the forefront of the light alloy industry.



MAGNUMINIUM

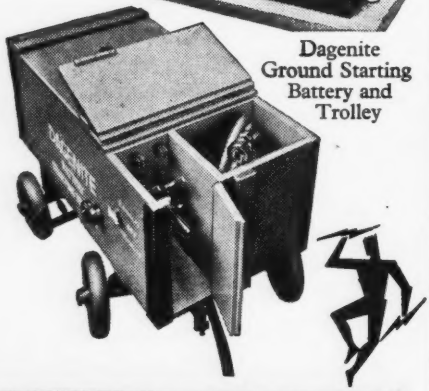
The Lightest Structural Metal

MAGNESIUM CASTINGS & PRODUCTS LTD · SLOUGH

SPECIALLY DESIGNED FOR SPECIAL JOBS

Dagenite
Non-Aerobatic type

Dagenite
Aerobatic type
(completely
unspillable)



Dagenite
Ground Starting
Battery and
Trolley

DAGENITE

ACCUMULATORS FOR AIRCRAFT

Write for Catalogue 102*a*

PETO & RADFORD

50 GROSVENOR GARDENS, LONDON, S.W.1. *SLOane* 7164

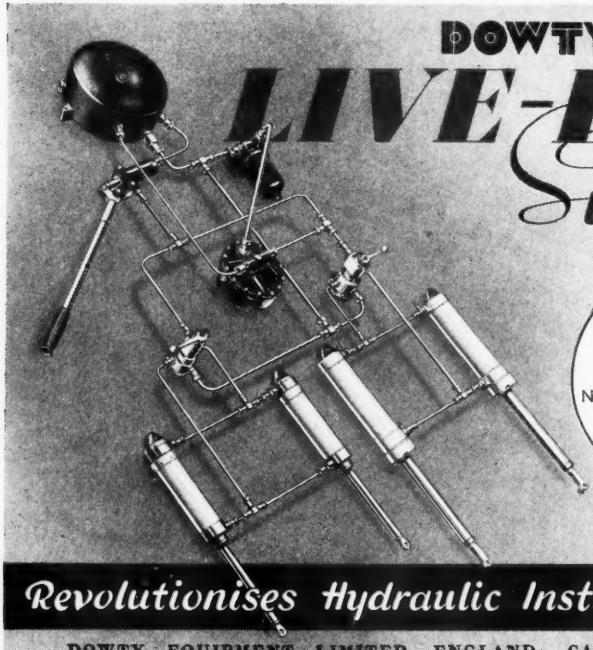
FR2e/42

The finest cored Solder in the world which is speeding up and increasing the efficiency of the War Effort. Firms engaged upon Government Contracts are invited to write for free samples.

MULTICORE SOLDERS LIMITED, BUSH HOUSE, W.C.2 'Phone Temp. Bar 5583/4

DOWTY

LIVE-LINE System



NO PRESSURE REGULATORS OR CUT-OUT VALVES

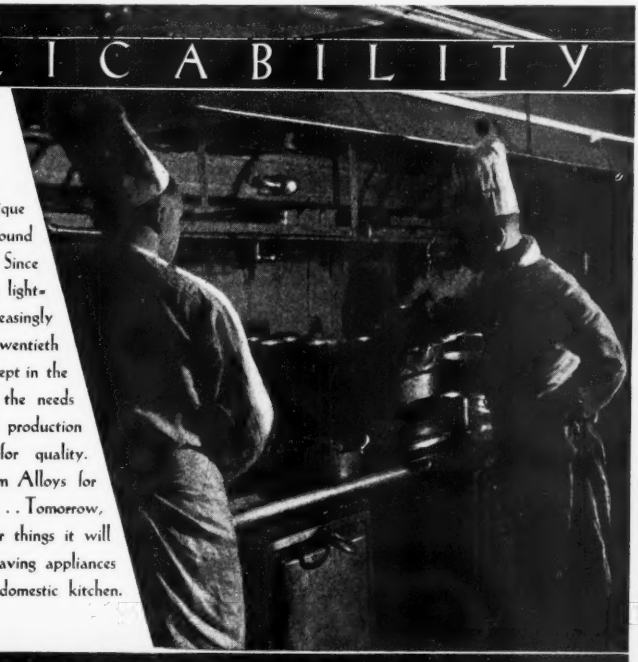
NO ACCUMULATORS

NO RELIEF VALVES

Revolutionises Hydraulic Installations

DOWTY EQUIPMENT LIMITED, ENGLAND; CANADA AND U.S.A.

APPLICABILITY



The first real appreciation of the unique qualities of Aluminium Alloys was found in the production of Kitchen utensils. Since that day the applications of these light-weight metals have become increasingly varied and diverse. In this twentieth century development we have kept in the van, adapting our product to the needs of its users, our system of production being standardised only for quality. For we produce Aluminium Alloys for every conceivable purpose . . . Tomorrow, amongst a thousand other things it will be for the new labour saving appliances for the canteen and domestic kitchen.

INTERNATIONAL ALLOYS LIMITED

SLOUGH. BUCKS.

IT is not mere facetiousness to draw a parallel between the work of the efficient G.P. and that accomplished by Linatex.

Linatex has a fine record of prevention and cure of the ill[s] machinery and indust[ry]. The ever-attend[ing] *abrasion.* [Linatex] are b[eing] [used] L[arge] re[sults] for[ming] ling[ering] basic [troubles].

EFFICIENT LINATEX SUBSTITUTES AVAILABLE

Linatex being 95% pure rubber is supplied for only the very highest priority applications. But Wilkinson research has not lagged behind. A full range of Linatex Substitutes for aircraft purposes is available. Each Substitute has been designed for its particular application by Linatex technical staff to give reliable performance. Linatex visiting engineers can advise you as to the most suitable substitute to replace your Linatex or other rubber components.

★

Our techn[ical] [staff] of qualified engineers will be pleased t[o] [discuss] you candidly whether Linatex can solve your problem more efficiently. Should you desire, they will gladly work with you, or train your own staff in using Linatex.

WILKINSON RUBBER LINATEX LTD.,
1-4 GREAT TOWER ST., E.C.3
Telephone: Mansion House 4333
Also in Canada, British Malaya, Australia, South Africa, U.S.A., etc.

A Basic Material for Industry

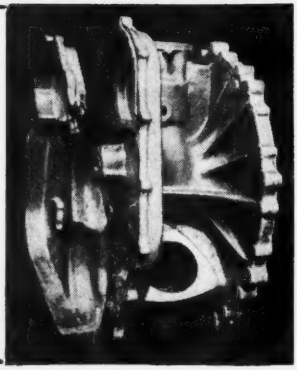
SEE-SEALING LINATEX 95% NATURAL RUBBER

A. V. ROE & CO., LIMITED *Branch of Hawker Siddeley Aircraft Co., Ltd.*

HIGH CLASS
ALUMINIUM SAND CASTING

Produced by—
AEROPLANE & MOTOR ALUMINIUM CASTINGS LTD.
WOOD LANE,
ERDINGTON,
BIRMINGHAM, 24

Suppliers of Aluminium and Magnesium Alloy Die and Sand Castings of every description.



Airscrew Hub Forging in Hiduminium R.R.56.

YESTERDAY it may have been so—but new developments in technique have made possible the production of high strength forgings of extremely complicated design, yet manufactured very close to final dimensions. They are economical both in material and in machining time.

The excellent workability of HIDUMINIUM Aluminium Alloys, together with their high strength/weight ratio, makes them essential materials for the post-war development of many industries. If you are at present using aluminium alloys, or if your future planning envisages their use—the Development Department will be pleased to send you further details.

Hiduminium

HIGH DUTY ALLOYS LIMITED · SLOUGH

SIR W. G.
ARMSTRONG WHITWORTH
AIRCRAFT LIMITED
BRANCH OF HAWKER SIDDELEY AIRCRAFT Co. Ltd.

Designers & Constructors of the Whitley Heavy Bomber

Leytonstone

JIG & TOOL Co. LTD

FULLY A.I.D APPROVED

SPECIALISTS IN
JIGS & FIXTURES
•
PRESS TOOLS
•
AIRCRAFT ASSEMBLY JIGS and REFERENCE GAUGES
•
HEAT TREATMENT carried out FOR THE TRADE

606 HIGH ROAD, LEYTON, LONDON, E.10. *Tel: Leytonstone 5022-3 Grams: Leytool, Leystone*

FACTS ABOUT

'PERSPEX'

REGD TRADE MARK

ACRYLIC RESIN SHEET

★ Retains its strength and does not become brittle at low temperature. At – 40°C. (24,000 ft.) its impact strength is greater than at ground level.

★ Its high clarity is retained in intense sun and ultra-violet light.

'PERSPEX' is the perfect transparent material for aircraft

I.C.I (PLASTICS) LTD.

(A subsidiary company of Imperial Chemical Industries Ltd.)

Sales Offices at: Mill Hill, London, N.W.7; Oldbury, near Birmingham; Alderley Edge, Cheshire; Bristol; York; Newcastle-on-Tyne; Glasgow; Belfast.

P105

what the Experimental Engineer says:

fit

GITSEALS PAT.

It is the meticulous care in design and manufacture by highly skilled operators in conjunction with continual checking, testing and proving at every stage that makes Gitseals so dependable. You will be following the lead of design executives and chief engineers and you will ensure longer life for all bearings. Gitseals can be supplied split and in other formations.

Manufacturers also of the
WESTON OIL RETAINER
suitable for any peripheral speed up to 500 feet per minute.

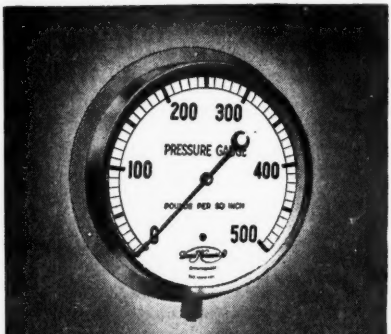
CHARLES WESTON & Co. Ltd., Torrington Avenue, COVENTRY

Telegrams: "Gitseals," Coventry. Telephone: Tilehill 66291-2
Branches: London, Gloucester, Manchester, Glasgow

A 294

Quality plus Design

SMITH'S
AIRCRAFT INSTRUMENTS



PRESSURE VACUUM AND COMPOUND GAUGES

David Harcourt Ltd

A.I.D. APPROVED FOR FUNCTIONAL TESTS
COVENTRY ROAD BIRMINGHAM 10
PHONE VICTORIA 1051 GRAMS LINKULA BIRMINGHAM

SHAWS

★ HOBSON INDUCTION PRESSURE (BOOST) CONTROLS

★ HOBSON-PENN AUTOMATIC MIXTURE CONTROLS

★ HOBSON PILOT'S COCKPIT CONTROLS

FOR ALL TYPES OF AERO ENGINES

HOBSON
(REGD. TRADE MARK)
Master Control Carburetters

H. M. HOBSON (AIRCRAFT & MOTOR COMPONENTS) LTD.

JABLO

WOODEN BLADES

The varying density of the JABLO Blade, from 1.30 near the root to 0.9 nearer to the tip, is made possible by the unique method of construction, and is in accordance with both strength and aerodynamic requirements.

JABLO PROPELLERS LIMITED · MANCHESTER 17

(R—B) JA 3a

AVIMO

PROGRESS *by* QUALITY

AERONAUTICAL EQUIPMENT

AVIMO LTD. TAUNTON. SOMERSET

THE SPECIALISED RUBBER COMPOSITIONS

Impervex

100% ON THE ADMIRALTY, WAR OFFICE AND AIR MINISTRY LISTS

PROVED EFFICIENCY

IMPERVIOUS to the action of Aviation Fuel, Hot Engine Oil, Di-Ethyl-Glycol, Acids, Alkalis, Salt Water, Steam and all Weather Conditions.

SUPPLIED IN Sheets, Gaskets, Washers, Mouldings, Strip, Tubing, Hose, etc., to any Specification and in any degree of resiliency.

IMPERVIA Ltd. 3, GROSVENOR GARDENS LONDON, S.W.1

'Phone and 'Grams : Victoria 6501

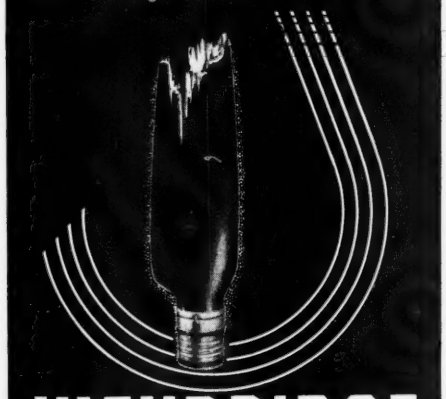
SUPPLIED TO LEADING ELECTRICAL, MECHANICAL & AIRCRAFT ENGINEERS

6417

PROPELLER SURGEONS

Extraordinary

As a surgeon's ski" restores the use of broken limbs, so the skill of our repairers restores broken and mutilated Weybridge Blades. They return to active service again and again, and are always perfectly shaped and balanced to give that efficiency upon which so much depends. There is no fatigue in wood!



WEYBRIDGE

AIRCRAFT PROPELLER

BLADES

THE ACE OF BLADES

THE AIRSCREW COMPANY LTD.

LODGE

playing their part

THE BEST PLUG IN THE WORLD FOR ALL AIRCRAFT ENGINES

Made by Lodge Plugs, Ltd., Rugby

CORK MANUFACTURING CO., LTD., South Chingford, London, E.4. Tel.: Silverthorn 2666 (7 lines)

(Associated with Flexo Plywood Industries Ltd.)

HIGH SPEED STEEL DRILLS

THOS FIRTH & JOHN BROWN LTD

THE JOURNAL
OF
THE ROYAL AERONAUTICAL SOCIETY

With which is Incorporated The Institution of Aeronautical Engineers.

CONTENTS FOR JANUARY, 1943

	PAGE
DYNAMIC STIFFNESS AND EFFECTIVE INERTIA METHODS IN MODERN TORSIONAL VIBRATION THEORY, BY R. G. MANLEY	5
THE ELECTRO-MECHANICAL ANALOGY IN OSCILLATION THEORY, BY R. G. MANLEY	22
REVIEWS	27
ABSTRACTS FROM THE SCIENTIFIC AND TECHNICAL PRESS	1
TITLES AND REFERENCES OF ARTICLES AND PAPERS	29

THE JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY *was founded in* 1897 *in succession to the* ANNUAL REPORTS.

For the opinions expressed in Papers that are signed or initialled the authors alone are responsible. None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless such is definitely stated to be the case.

The Journal is published monthly at the Offices of the Society, 4, Hamilton Place, Piccadilly, London, W.1.

Subscriptions per annum, £3 . 3 . 0 post free; Single Numbers 5/- or 5/3 post free.

All communications for publication in the Journal, or on general matters affecting the Society should be addressed to—

The Secretary and Editor,
J. Laurence Pritchard,
4, Hamilton Place, W.1.

The Society's Bankers are Messrs. Coutts & Co.

All communications respecting Advertisement matters should be addressed to—
The Advertisement Manager, Journal of the Royal Aeronautical Society,
4, Hamilton Place, W.1.
Telephone : Grosvenor 3515-3516-3517. Telegraphic Address : "Didaskolos, Audley, London."
Or to— T. G. Scott & Son, Ltd., Talbot House, Arundel House, London, W.C.2.
Telephone : Temple Bar 1942/3.

YES
Telephones—Armley
38234 & 38291 2 lines

Y. E. S. LTD.

YES
Telegraphic Address—
"YES, LEEDS"

PHOSPHOR BRONZE AND GUNMETAL BEARINGS & BUSHES

OF THE HIGHEST QUALITY . . . . CAST BY THE
"EATONIA" (WATER-COOLED) PROCESS

CASTINGS AND BARS AND FINISHED MACHINED PARTS SUPPLIED TO THE LEADING
AEROPLANE ENGINE MAKERS MOTOR VEHICLE ENGINE MANUFACTURERS
MARINE ENGINE BUILDERS

Contractors to all Government Departments

YORKSHIRE ENGINEERING SUPPLIES LIMITED
BRONZE FOUNDRIES, WORTLEY, LEEDS, 12

Aircraft Engineering

FOUNDED 1929

The Technical and Scientific Aeronautical Monthly
Edited by Lt.-Col. W. Lockwood Marsh, O.B.E., F.R.Ae.S., M.S.A.E., F.I.Ae.S.

Single Copies **2s. 3d.** post free. Ordinary Subscription : **26s. 0d.** per annum, post free
SPECIAL SUBSCRIPTION RATE *For Students, Associates and Graduates of the Society* : **21s. 0d.** per annum, post free

PRINCIPAL JANUARY CONTENTS

The Junkers Ju 211 Engine.
Two-Spar Wing-Stress Analysis I. W. J. Goodey.
Air-Line Engineering Management VI. Ivor Lusty.
Standardization of Aero-Engine Components. G. Carvelli.

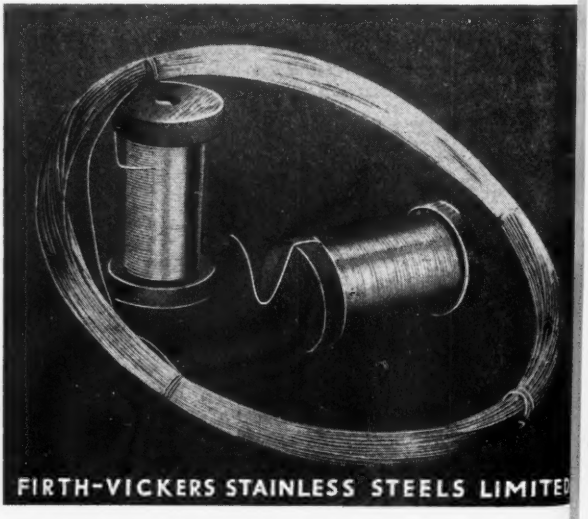
BUNHILL PUBLICATIONS LIMITED, 12 Bloomsbury Square, London, W.C.1

(55)

FIRTH VICKERS
"Staybrite"
REG. TRADE MARK
SUPER RUSTLESS STEEL

The importance of strength plus resistance to corrosion can hardly be overestimated in the case of wire. Inaccessibility and difficulty of replacement in key situations are often found, and wire is one of the forms of "Staybrite" Steel in which the answer to many hitherto difficult problems is provided.

Owing to the large demand, we are at present greatly restricted as regards the purposes for which this steel can be supplied.



THE ROYAL AERONAUTICAL SOCIETY

(Founded 1866*)*

with which is incorporated the Institution of Aeronautical Engineers

VOLUME XLVII

MONTHLY NOTICES,

JANUARY, 1943.

New Year Message.

The President and Council offer their good wishes for the coming year to all members, here or overseas, with the hope that they will come safely through the struggle which is ahead.

Council Election.

Attention is drawn to the following Rules for election of Council. Nominations for Council should be received by March 3rd, 1943. Nomination forms may be obtained from the Secretary. Attention is drawn to possible delays in the post at the present time.

RULE 17.—One half of the Council (excluding the President and the immediate Past-President if a member of Council) shall retire annually. The members who shall retire shall be those longest in office, except as provided in Rule 77. Retiring members of Council who have served for two terms in succession (four years) shall not be eligible for re-election until the next annual election, when they will be eligible.

RULE 72.—The composition of the Council shall be as follows:—

(*a*) Not less than sixteen shall be elected from the technical or scientific grades of the R.Ae.S., that is to say, from among the Fellows, Associate Fellows, Associates and Graduates.

(*b*) Of these sixteen, four at least shall be Fellows.

RULE 73.—Nominations of candidates for election to the Council must be received by the Secretary not less than twenty-one days before the Annual General Meeting, with an intimation in writing by the Candidates that they are willing to serve. Nominations must be signed by one supporter and two seconders, who must be each entitled to vote in the R.Ae.S.

Endowment Fund.

The Council acknowledge with thanks contributions to the Endowment Fund from Warrant Officer J. V. Saunders, Associate Fellow, and Mr. J. C. Stevenson, Associate Fellow.

Advertisements.

The fact that goods made of raw materials in short supply owing to war conditions, are advertised in the Journal, should not be taken as an indication that they are necessarily available for export.

Annual Subscriptions.

Members are reminded that their subscriptions are due on January 1st, 1943. The following are the current rates:—

	Home. £ s. d.	Abroad. £ s. d.
Founder Members	2 2 0	2 2 0
Fellows	4 4 0	3 3 0
Associate Fellows	3 3 0	2 2 0
Associates	*2 2 0	*2 2 0
Graduates (age 21-25)	2 2 0	2 2 0
,, (age 26-28)	2 12 6	2 12 6
Students	1 1 0	1 1 0
Companions	2 2 0	2 2 0

* £1 1s. 0d. without JOURNAL.

All those members who have relinquished their Journal have been notified of the reduction in their subscriptions due on January 1st, 1943.

Election of Members.

Associate Fellows.—Leslie William Andrews (from Graduate), Geoffrey Burton Crosby, Leonard Sidney Greenland, Archibald Louis Harriss, Andrew Keith Hunter (from Graduate), Gustav Andrzeji Mokrzycki, William Poole.

Associates.—Christopher Henry George Bartlett Barnes, William George Brinn, Robert George Burgess, Albert Leonard Cattermull (from Student), Frank Dennis Collyer, Leslie William Dixon, George Christie Hindmarch, George Frederick Mason, Stanley Francis Potter (from Companion), Isaac Eyton Roberts, William Edwin John Smith, Denis Heaton Thorns.

Graduates.—David Cardwell, Leslie Arthur Hopkins, Raghunath Shamrao Khot (from Student), Alfred Stanley Taylor (from Student), Arthur Frederick Toe (from Student), Cyril Francis Trigg (from Student).

Companions.—Donald Diamond (from Student), Paul Steiner, Joseph Douglas Whitell (from Student).

Students.—John William Adams, Connell Frederick Bee, David Glenn Boadle, Colin Morris Britland, Maurice Reginald Chantrill, James Frederick Creckendon, Mansel Edward David, William Devon Doble, Kenneth Essex-Crosby, Austin Roy Fox, Norman Grosvenor Garner, Arthur Sydney Garwood, William Bell Gosney, Joseph Cyril Guttridge, William Geoffrey Heath, Charles Roger Heaton, Arthur William George Hersee, Dennis Stanley Houghton, Rolf Frederick Marshall, William Stanley Donald Marshall, William Henry Mayall, Jiri Mraz, Bernard George Neal, Harold George Newbigin, Edward James Oag, Ian Philip Osler, Thomas Stuart Parramore, William Frodsham Pritchard, Elmer Protheroe, Gordon Rymer, Eric Smith, Henry Lambert Garbutt Sunley, Peter George Tranter, Harry Tyler, Gregory Frank Webb, Robert Murdock Wilson.

Graduates' and Students' Section.

Members of the Graduates' and Students' Section are invited to the following lectures arranged by the Graduates' and Students' Section of the Institution of Automobile Engineers, 12, Hobart Place, S.W.1:—

Sunday, January 10th, 1943, at 2.30 p.m.—Informal Talk by Mr. L. Pomeroy on "The Aerodynamic Car."

Sunday, February 14th, 1943, at 2.30 p.m.—Paper by Mr. E. S. Crillo on "Drawing Office Practice."

Sunday, March 14th, 1943, at 3.0 p.m.—Paper by Mr. J. H. Millward on "The Gearbox."

Sunday, April 11th, 1943, at 3.0 p.m.—Informal Talk by Mr. L. Mantell on "The Practical Aspects of Thermal Efficiency."

Sunday, May 16th, 1943, at 3.0 p.m.—Informal Talk by Mr. D. H. Bramley on "Works Organisation."

Additions to the Library.

Pamphlets in italics with location reference following in brackets.
Books marked * or ** may not be taken out on loan.

B.a.272.—Sea-Flyers. By C. G. Grey. Faber and Faber. 1942. 7/6.

*D.a.C.19.—Civil Aviation Statistics (Canada). March, April, May, 1942. Dominion Bureau of Statistics, Ottawa.

*EE.b.67 and 68.—*Ministry of Aircraft Production, Official Reports: B.M.W. 132K (1) and 132N Engines; Bramo Fafnir 323-P-1 Engine and Installation.* 1942. (Y.3.iii, 9/10.)

*G.b.44.—S.B.A.C. Standards Handbook, Vol. III. Society of British Aircraft Constructors. 1942.

G.a.A.84.—*Northern Aluminium Co., Ltd., Research and Development Bulletin. The Mechanical Testing of Aluminium and Aluminium Alloy Products and its Bearing on Working Properties.* 1942.

G.e.20.—*Beryllium-Copper. By L. David. Beryllium Smelting Co.* 1942. (PG.3.b.10.)

G.e.E.57.—*Lecture on Rust-, Acid- and Heat-Resisting Steels. By Dr. W. H. Hatfield. (Given before the Institute of Chemistry, May 20, 1942. Reprint.)* (PG.6/18.)

I.a.30.—Aircraft Calculations. By S. A. Walling and J. C. Hill. (2nd edition of "Aircraft Mathematics.") Cambridge University Press. 1942. 3/-.

L.d.81.—Numerical Examples in Elementary Air Navigation. By G. K. Clatworthy. (2nd Ed.) Sir Isaac Pitman and Sons. 1942. 3/6.

L.d.82.—The Complete Air Navigator. By D. T. C. Bennett. (4th Ed.) Sir Isaac Pitman and Sons. 1942. 15/-.

L.d.83.—Astronomical Navigation. By W. M. Smart. Longmans, Green and Co. 1942. 5/-.

M.c.61.—Aircraft Radio. By D. Hay Surgeoner. Sir Isaac Pitman and Sons. 1942. 15/-.

N.a.74.—Sub-Atomic Physics. By Herbert Dingle. Thos. Nelson and Sons. 1942. 5/-.

N.a.75.—Abstracts of Dissertations approved for the Ph.D., M.Sc. and M.Litt. Degrees in the University of Cambridge (1940-1941). University Press, Cambridge. 1942.

*Q.a.145.—Handbook of British Standards, July, 1942 (with October Supplement). British Standards Institution. 1942. 1/6.

*Q.a.146.—*A.S.L.I.B. War-Time Guides to British Sources of Specialised Information. No. 4. Scientific and Technical Periodicals received from the U.S.S.R.*

**R.f.81.—America Fledges Wings. (The History of the Daniel Guggenheim Fund for the Promotion of Aeronautics.) By Reginald M. Cleveland. Pitman Publishing Corp., N.Y. $4.50. 1942.

S.b.116.—What's the Gen? (R.A.F. Slang illustrated.) By "H.W." John Crowther. 1942. 1/3.

S.d.79.—Air Defence and the Civil Population. By H. Montgomery Hyde and G. R. Falkiner-Nuttall. Cresset Press. 1938. 3/6.

T.a.102.—Cæsar of the Skies. (The Official Life of Sir Charles Kingsford-Smith.) By Beau Sheil. Cassell and Co. 1937. 8/6.

TT.a.39.—Ssh! Gremlins! By "H.W." John Crowther. 1942. 2/-.

*UU.c.—*National Advisory Committee for Aeronautics (U.S.A.): Technical Memoranda*:—

No. 1030. The Compressible Potential Flow Past Elliptic Symmetrical Cylinders at Zero Angle of Attack and with no Circulation. (From Luftfahrtforschung, Sept., 1941.) W. Hantzsche and H. Wendt.

No. 1032. Heat Transfer in the Turbulent Boundary Layer of a Compressible Gas at High Speeds. F. Frankl; and Friction in the Turbulent Boundary Layer of a Compressible Gas at High Speeds. By F. Frankl and V. Voishel. (C.A.H.I. No. 240, 1945.)

*WB.21.—Journal and Proceedings of the Royal Society of New South Wales. Vol. LXXV, Part IV (1941).

J. Laurence Pritchard, *Secretary and Editor.*

" DYNAMIC STIFFNESS AND EFFECTIVE INERTIA METHODS IN MODERN TORSIONAL VIBRATION THEORY."

By R. G. MANLEY (Student Member).

INTRODUCTION.

The chief aim of the vibration engineer is to prevent the occurrence of structural failure due to excessive vibration. A secondary aim of only slightly less importance is to ensure comfort for personnel by the elimination of physiologically unpleasant vibrations; this consideration is a vital factor in the effective design of operational aircraft. The technique of vibration research is largely a matter of detecting and avoiding resonant conditions, and any method of predicting natural frequencies by calculation is welcome if only because it offers the possibility of reducing the practical testing programme.

Torsional motion in the engine/propeller system gives rise to one of the most troublesome forms of vibration encountered in aircraft, but fortunately this type of oscillation is easily studied with the aid of the concepts of dynamic stiffness (" mechanical impedance ") and effective inertia. The latter has been utilised, in its simple form, for many years; it was not until the dynamic stiffness characteristics of torsional systems had been investigated, however, and the fundamental relation between the two quantities derived, that the technique of calculating the torsional natural frequencies of aircraft power-plants was brought to maturity and could be applied to the highly developed modern combinations.

It is not clear to whom is due the credit of discovering the properties of dynamic stiffness. The quantity is loosely analogous to the impedance of electric circuits, and the author has described elsewhere (reference 11) a precise analogy between mechanical vibrating systems and electric circuits; the term " mechanical impedance " was used in 1919, and the method of torsional vibration analysis based on this quantity has been utilised by the de Havilland Aircraft Company since 1939, but Biot (reference 10) was the first to give effective publicity to the advantages of the method. Since the publication of his paper in 1940 the British aero-engine manufacturers have almost universally adopted the technique.

In this paper an attempt is made to give a brief but comprehensive discussion of these concepts so as to provide in one place a key to the complex problem of determining the resonant frequencies of engine/propeller systems; different aspects of the problem have been discussed in widely scattered references, and the present work is intended to unify these partial solutions into a rational whole.

SUMMARY.

Dynamic stiffness and effective inertia are formally defined, and the application of the concepts to linear and damped systems indicated. Confining further attention to undamped torsional motion, since in practice the effects of damping forces cannot be determined quantitatively, the problem of determining natural frequencies is reduced to that of finding the frequencies for which certain quantities, which are functions of the dynamic constants, are zero. Methods of determining these quantities are described, including the practical method of measuring the dynamic stiffness of propellers and methods of calculating the same quantity for engines, with or without the complications of heavy shafts or gearing. Tuning inertias and systems are discussed, and the text concludes with a brief description of five applications where the method has been found to be particularly

effective. In the appendix is given a proof of the theorem that the number of real distinct natural frequencies of a torsional system is equal to the number of degrees of vibrational freedom.

I. Formal Definition of Dynamic Stiffness and Effective Inertia.

Confining present attention to undamped torsional systems, consider such a system performing torsional vibrations under the action of a single sinusoidal torque applied at a point P. Let the torque be $T=T_0 \sin wt$, and let the resulting angular displacement at P be $x=x_0 \sin wt$,* so that the frequency F of vibration is given by $F=w/2\pi$ C.P.S., w being the phase velocity in radians per second. The ratio $T/x=T_0/x_0$ is defined to be the *dynamic stiffness* of the system at P for the frequency F, and is denoted by Z_p, or more simply by Z if there is no ambiguity. Since the static stiffness of a shaft is defined to be the amplitude of transmitted torque required to produce unit twist in the shaft, Z is seen to be a precise dynamic analogue of the static stiffness, and the name " dynamic stiffness " is therefore to be preferred to certain other terms such as " mechanical impedance " and " dynamic modulus."

$$Z_p=T/x=T_0/x_0 \quad . \quad . \quad . \quad . \quad . \quad (1.1)$$

or, in words, the dynamic stiffness for a frequency F at a point P in a torsional system is the amplitude of the torque which must be applied at P to produce at P an angular displacement of unit amplitude and frequency F. The reciprocal $1/Z_p$ is called the *dynamic flexibility* (or " admittance "), and is a measure of the response of the system to a single sinusoidal torque applied at P. Thus if the dynamic stiffness is greater at a frequency F_1 than it is at a frequency F_2, so that the flexibility is less at F_1 than at F_2, the displacement-amplitude for a given applied torque at the frequency F_1 will be less than at F_2.

It is easily seen that Z is a function of the frequency; for the natural frequencies of the system are those frequencies at which a sinusoidal motion can be maintained, in the absence of damping forces, without any external applied torque, so that for these frequencies, and these frequencies alone, the dynamic stiffness at any point in the system is zero (see note at end of section III).

Similarly, if the acceleration at P is $a=a_0 \sin wt$, the *effective inertia* at P for the frequency $F=w/2\pi$ C.P.S. is defined as the ratio $T_0/a_0=T/a$, and is denoted by J_e.

$$J_e=T/a=T_0/a_0 \quad . \quad . \quad . \quad . \quad . \quad (1.2)$$

Since $a=\ddot{x}=-w^2x_0 \sin wt$, there is a fundamental relation between Z and J_e for any value of w, which can be expressed thus:—

$$Z=-J_e w^2 \quad . \quad . \quad . \quad . \quad . \quad (1.3)$$

II. Linear and Damped Systems.

Precisely analogous expressions can be derived for the dynamic stiffness and effective mass of linear systems of the conventional spring-plus-mass variety. In the engineer's system of measurement the appropriate units for torsional motion are lbs. ins./radian for dynamic stiffness and lbs. ins. secs.^{2} for effective inertia, and in linear motion the units are lbs./in. for dynamic stiffness and lbs. ins.$^{-1}$ secs.^{2} for effective mass.

The concept can also be extended to systems including damping forces. In such cases the dynamic stiffness cannot be zero at any non-zero frequency, since an external applied torque is always required to supply the energy dissipated. The extension of the theory to damped systems has been described elsewhere (reference 1); it is sufficient to state here that the displacement corresponding to a torque $T=T_0 \sin wt$ is of the form $x=x_0 \sin (wt+\phi)$, where ϕ is non-zero, and that the dynamic stiffness ratio T_0/x_0 is a minimum for resonant frequencies.

* Where x_0 may have either sign, according as the displacement is in-phase, or 180° out-of-phase, with the torque.

The characteristics of damped systems are of little importance in theoretical studies, except when it is required to calculate finite displacement-amplitudes and stresses; for such calculations the damping forces cannot be neglected, but with modern instruments and testing technique it is seldom necessary to rely upon calculated amplitude and stress values. In any case, two considerations limit the usefulness of calculations involving damping forces: (i) It is extremely difficult to determine the magnitude of the damping forces which are present in a system, and (ii) the theoretical assumption that such forces are proportional to velocity, which is necessary in order to obtain a mathematical solution, is not justified in practice. The conclusions drawn from any theoretical analysis based on this assumption can only be used qualitatively; *i.e.*, the general effects of damping upon dynamic stiffness characteristics can be determined (see reference 1), but precise quantitative conclusions cannot be drawn. Actual strain-gauge tests provide the only reliable method of determining vibration stresses.

It is by a consideration of the dynamic stiffness properties of damped systems that it is possible to establish an analogy between mechanical vibrating systems and electric circuits. This analogy is described elsewhere (reference 11).

III. Natural Frequencies and Coupled Systems.

The determination of the natural frequencies of an undamped system is equivalent to finding those frequencies for which the dynamic stiffness or effective inertia at any point is zero. Consider the system shown in Fig. 1*a*, consisting of two sub-systems A and B coupled at P. Let the dynamic stiffnesses of the two systems separately at P be Z_A and Z_B, and let the torques at P required to produce an angular displacement x at P be T_A and T_B in the same sense. Then

$$T_A = Z_A x, \quad T_B = Z_B x.$$

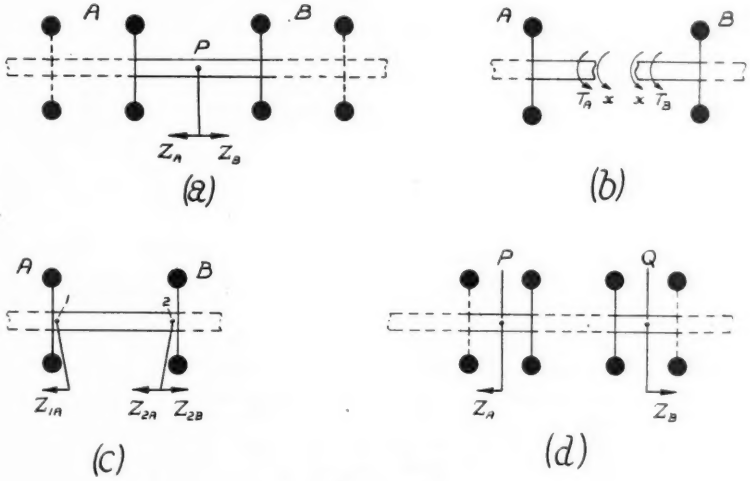


Fig. 1.
Twice full size.

The natural frequencies of the combined system are those at which the total applied torque $(T_A + T_B)$ required to maintain the finite displacement x is zero. Hence

$$Z_A + Z_B = 0 \text{ at natural frequencies} \quad . \quad . \quad . \quad (3.1)$$

Methods of determining the dynamic stiffnesses of systems are described below (section IV). If the dynamic stiffnesses of two coupled systems such as A and B in Fig. 1 are known functions of the frequency, the natural frequencies of the combined system are easily determined by (3.1). It is only necessary to plot Z_A and $-Z_B$ on the same axes and to note the frequencies corresponding to the intersections of the two curves (reference 2). This procedure is of immense utility in the treatment of vibration troubles in aero-engine/propeller systems, wherein it is possible to calculate the dynamic stiffnesses of the engine and propeller with reference to the propeller hub; the method of intersections then determines natural frequencies, and the effects of modifying either part of the system are easily ascertained.

By reason of the relation (1.3) the condition (3.1) for natural frequencies can be expressed in terms of effective inertias. Thus if J_A and J_B are the effective inertias of A and B at P,

$$J_A + J_B = 0 \text{ at natural frequencies} \quad . \quad . \quad . \quad (3.2)$$

Consider now the system shown in Fig. 1*c*, consisting of two sub-systems A and B coupled by the shaft (1)-(2). Let the dynamic stiffnesses of A at (1) and B at (2) be Z_{1A} and Z_{2B} as shown, and let the displacements at (1) and (2) be x_1 and x_2. Suppose that the part B is removed, and a torque T is applied at (2). The shaft transmits this torque to (1), and if K is the stiffness of the shaft the following equations are obtained:—

$$T = K(x_2 - x_1)$$
$$T = Z_{1A} x_1$$
$$T = Z_{2A} x_2$$

where Z_{2A} is the dynamic stiffness of A-plus-shaft at (2). From these equations it is found that

$$1/Z_{2A} = 1/Z_{1A} + 1/K \quad . \quad . \quad . \quad . \quad (3.3)$$

The condition for natural frequencies of the combined system is $Z_{2A} + Z_{2B} = 0$, *i.e.*,

$$-1/Z_{2B} = 1/Z_{1A} + 1/K \quad . \quad . \quad . \quad . \quad (3.4)$$

The equation (3.2) is convenient when the points in the sub-systems at which the dynamic stiffnesses are known coincide in the combined system, but (3.4) is easier to apply when these points are separated by a shaft as in Fig. 1*c*. For such cases it would also seem advantageous to make use of the reciprocal quantity (dynamic flexibility), but in actual practice there is little to choose between the two quantities Z and $1/Z$, and employment of one or the other is merely a matter of personal preference.

NOTE.

It is not immediately apparent that if the dynamic stiffness at a point P in a system is zero, then the dynamic stiffness at any other point Q is also zero. The result can however be proved. In the system of Fig. 1*d* let Z_A and Z_B be the dynamic stiffnesses of the systems to the left of P and to the right of Q respectively; and let the transference formula connecting dynamic stiffness at P and Q be

$$Z_P = f(Z_Q) \quad . \quad . \quad . \quad . \quad . \quad (3.5)$$

so that Z_Q is given in terms of Z_P by the inverse functional form

$$Z_Q = f^{-1}(Z_P) \quad . \quad . \quad . \quad . \quad . \quad (3.6)$$

where, no matter what the value of y may be,

$$f^{-1}[f(y)] = y \quad . \quad . \quad . \quad . \quad . \quad (3.7)$$

Now, if the dynamic stiffness at P is zero,

$$Z_A + f(Z_B) = 0$$

so that

$$f^{-1}(Z_A) + f^{-1}[f(Z_B)] = 0$$

This last equation, by reason of (3.7), reduces to

$$Z_B + f^{-1}(Z_A) = 0$$

i.e., $Z_Q = 0$.

IV. Determination of Dynamic Stiffness.

(*a*) Practical Method.

Torsional dynamic stiffness can be measured practically by means of a torsional vibrator. This apparatus consists of a shaft, provided at one end with a means for attaching the torsional system whose dynamic stiffness it is required to find, and driven from the other end by an exciter unit which is designed to apply a sinusoidal torque of variable frequency. Built into the shaft is a torque-gauge which measures the twist in a certain length of the shaft and so indicates the transmitted torque, and a torsiograph is attached at the coupling point to measure the torsional displacement amplitude. The required dynamic stiffness can be obtained directly from these two sets of observations over a frequency-range limited only by the design of the vibrator. A description of the theory and design of the exciter unit is given in reference 3.

The method as described is admirably suited to the task of determining the dynamic stiffness of aircraft-propellers, since the hub is easily mounted on an adaptor and attached to the exciter-shaft, and a torsiograph can be fixed on the front of the hub; the hub is sufficiently rigid to render it immaterial whether the displacement at front or rear of the propeller is measured. The method is not readily applicable to aero engines, but fortunately the torsional characteristics of these structures can easily be calculated to the required degree of accuracy (see below).

(*b*) Methods of Calculation.

There are three methods of calculating the dynamic stiffness of systems consisting of concentrated inertias and effectively light shafts, *i.e.*, in which there is a number of inertias, rigid in comparison with the shafts connecting them, and the shafts have polar inertias which are small compared with the concentrated inertias. The first two methods to be described are useful in certain applications, while the third is the " standard " frequency-tabulation or torque-summation method.

The determination of the inertias and shaft stiffnesses of the idealised system representing the actual engine is a matter which lies outside the scope of the present paper; a very full description of the methods employed is given by Ker Wilson (reference 12).

(§1) Partial Inertias; Dynamic Stiffness Curves.

Consider first the two-mass system shown in Fig. 2. If the inertias are measured in lbs. ins. secs.^{2} and the shaft stiffness in lbs. ins./radian, the phase velocity w in radians/second at the natural frequency is given by

$$w^2 = K(J_1 + J_2)/J_1 J_2.$$

Suppose the inertia J_2 is divided into two parts, H_2 and I_2, the *partial inertia* H_2 being such that the natural frequency of the *partial system* $J_1 - (K) - H_2$ is $w/2\pi$ C.P.S. Then

$$w^2 = K(J_1 + H_2)/J_1 H_2$$

and

$$\left.\begin{aligned} H_2 &= KJ_1/(w^2 J_1 - K) \\ I_2 &= J_2 - H_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4.1)$$

The system can be regarded as consisting of two parts: (i) The partial system $J_1 - (K) - H_2$ which is vibrating at its own natural frequency, and thus has zero effective inertia at any point, and (ii) the remaining inertia I_2. The effective inertia of the system at J_2 is therefore I_2. The condition $I_2 = 0$ leads back

immediately to the normal expression for the natural frequency of a two-mass system, but the method admits of extension to multi-mass systems. For example, in the system shown in Fig. 3*a* let

$$J_2 = H_2 + I_2, \quad J_3 = H_3 + I_3, \quad J_4 = H_4 + I_4.$$

H_2 is given by (4.1), and at the frequency 13,500 C.P.M. (for which $w^2 = 2 \times 10^6$ radians^2/secs.^{2}),

$$I_2 = J_2 - H_2 = 2.333.$$

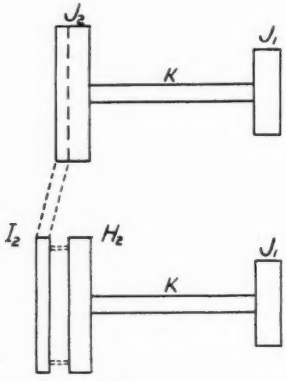
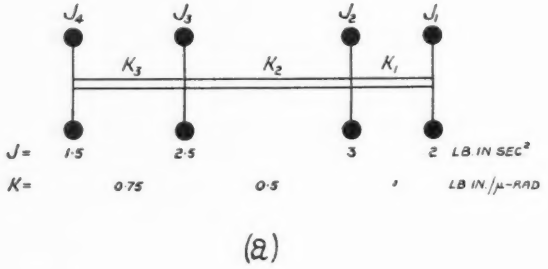
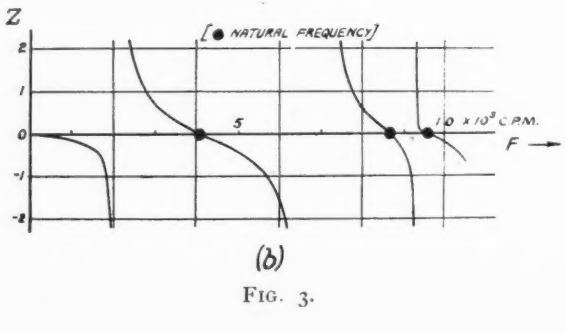


FIG. 2.



(a)



(b)

FIG. 3.

(The stiffness values quoted in the diagram are in the units: lbs. ins./microradian, and must be multiplied by 10^6 before being used in the formula 4.1.) Now choose H_3 so that the partial system $I_2-(K_2)-H_3$ has a natural frequency of 13,500 C.P.M. (*i.e.*, w^2 has the same value as for the partial system $J_1-(K_1)-H_2$). Then

$$H_3=K_2I_2/(w^2I_2-K_2)=0.280$$
$$I_3=J_3-H_3=2.220,$$

and similarly $I_4=1.049$.

The system can be regarded as consisting of two parts: (i) The partial system $J_1-J_2-J_3-H_4$, which is vibrating at one of its natural frequencies, and thus has zero effective inertia at any point, and (ii) the remaining inertia I_4. The effective inertia of the system at J_4 for 13,500 C.P.M. is therefore 1.049 lbs. in. secs.^{2}, and the dynamic stiffness is given by (1.3) as -2.098×10^6 lbs. ins./radian, or -2.098 lbs. ins./microradian.

In a similar manner the dynamic stiffness at J_4 can be found for other frequencies, and a curve drawn of Z against frequency. The approximate form of this curve, for the system of Fig. 3*a*, is shown in Fig. 3*b*. In practice the computation is readily tabulated in a concise form, and slide-rule accuracy is usually sufficient, although access to an automatic calculating machine naturally facilitates speedy working.

Apart from numerical applications, the method of analysis just described enables the two fundamental principles of the theory of multi-mass systems to be demonstrated, namely, (i) that any pair of adjacent inertias move either in-phase or 180° out-of-phase in the absence of damping forces, and (ii) that the resonant frequencies of lightly-damped systems are very nearly equal to the natural frequencies calculated on the assumption of zero damping. The proofs are given in reference 4.

The shape of the curve in Fig. 3*b* is typical of effective inertia or dynamic stiffness curves, in so far as zero values and infinite discontinuities occur alternately as the frequency is gradually increased from zero to a large positive value. Biot has pointed out (reference 10) that this property is inherent in the mathematical nature of the quantities. It is also worth noting that the curve is determined completely when the frequencies for which Z is zero and those for which Z is discontinuous, and the total inertia of the system, are known. The system of Fig. 3*a*, for example, possesses seven dynamic constants (four inertias and three stiffnesses) which can be varied, and the dynamic stiffness curve is therefore completely specified by seven independent quantities; these are conveniently taken to be the three values of F for which Z is zero, the three for which Z is discontinuous, and a scale-factor for Z. The dynamic stiffness curve approximates to the form $Z=-J_4w^2$ for very high frequencies; the effective inertia curve approximates to the asymptotic line $J_e=J_4$ for very high frequencies, and to the line $J_e=\Sigma J=J_1+J_2+J_3+J_4$ for very low frequencies. Since this last quantity is independent of the reference point it is conveniently taken to determine the scale-factor. Biot (*loc. cit.*) has given a formula expressing the dynamic stiffness at any frequency in terms of these quantities.

As a point of practical procedure it is worth while mentioning that if one or two points on either side of a discontinuity are known, the precise value of F for which Z becomes discontinuous can easily be determined, since this is the same condition as making $1/Z$ zero; if $1/Z$ is plotted in the immediate neighbourhood of the discontinuity in Z, the corresponding frequency value is indicated quite accurately, as the curve of $1/Z$ is continuous in this region.

(§2) Transference Formulæ.

In Fig. 4 the dynamic stiffness Z_1 of the system to the right of the inertia J is known, and it is required to find the dynamic stiffness Z_2 of the system to

the right of the point (2) which is immediately to the left of J. Since the inertia is supposed rigid, the displacement at (2) is the same as that at (1); let it be x. The torque T_1 which must be applied at (1) to produce this displacement is $Z_1 x$, while the torque T_2 which must be applied at (2) to produce the displacement x is $T_1 - Jw^2x$, as the differential torque $T_2 - T_1$ maintains the acceleration $-w^2x$ of the inertia J. Since $Z_2 = T_2/x$ the relation follows:—

$$Z_2 = Z_1 - Jw^2 \quad . \quad . \quad . \quad . \quad . \quad (4.2)$$

The corresponding formula for transferring the reference point through a shaft of stiffness K has already been given (3.3) and is repeated here for convenience:—

$$1/Z_2 = 1/Z_1 + 1/K \quad . \quad . \quad . \quad . \quad . \quad (4.3)$$

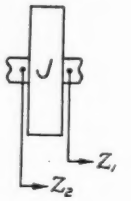


FIG. 4.

By alternate use of the transference formulæ the dynamic stiffness of a multi-mass system is obtained. Applying the method to the system of Fig. 3*a*, denoting dynamic stiffnesses at points to the right of inertias by Z_1, etc., and those at points to the left of inertias by Z_1', etc., and for convenience expressing all stiffnesses in lbs. ins./microradian, the working is as follows:—

At 13,500 C.P.M. ($w^2 = 2 \times 10^6$), since $Z_1 = 0$ and $J_1 = 2$, $Z_1' = -4$; but $Z_2 = K_1 Z_1'/(K_1 + Z_1')$, hence $Z_2 = -4/-3 = 1.333$. Similarly, $Z_2' = -4.667$, $Z_3 = -2.333/-4.167 = 0.560$, and eventually it is found that $Z_4' = -2.098$ lbs. ins./microradian as before.

Gear-steps, such as the reduction gear between an aero-engine and the propeller, can be dealt with by this method (see §5 below), and the method provides the only satisfactory means of determining the natural frequencies of systems including heavy shafts (see §4 below).

(§3) FREQUENCY (TORQUE SUMMATION) TABLES.

The standard method of determining the dynamic stiffness of multi-mass systems is by means of frequency tables, or "torque summation." For simplicity consider the system already investigated, Fig. 3*a*, and suppose it to be vibrating under the action of a torque T applied at J_4. Let the displacements of the inertias be x_1, etc., and let the torques transmitted in the shafts be T_1, etc. Then $T_1 = K_1(x_2 - x_1)$, and since this torque is maintaining the acceleration $-w^2x_1$ of the inertia J_1, $T_1 = -J_1w^2x$. Denoting the twist $(x_1 - x_2)$ in the shaft K_1 by d_1,

$$x_2 = x_1 - d_1 = x_1 - J_1w^2x_1/K_1.$$

Similarly

$$T_2 = K_2(x_3 - x_2), \text{ and } T_2 - T_1 = -J_2w^2x_2,$$

so that

$$x_3 = x_2 - d_2 = x_2 - (J_1w^2x_1 + J_2w^2x_2)/K_2.$$

Proceeding in this manner the displacement x_4 can be calculated, and the total torque which must be applied at J_4 to maintain this displacement is

$$T = -w^2(J_1x_1 + J_2x_2 + J_3x_3 + J_4x_4).$$

The dynamic stiffness Z at J_4 is then given by $Z = T/x_4$.

TABLE I.

FREQUENCY TABULATION FOR SYSTEM OF FIG. 3*a*.

$w^2 = 2 \times 10^6$.

Inertia ref.	J	Jw^2	x	Jw^2x	ΣJw^2x	K	d
1	2	4	1	4	4	1	4
2	3	6	−3	−18	−14	0.5	−28
3	2.5	5	25	125	111	0.75	148
4	1.5	3	−123	−369	−258	—	—

$Z = -\Sigma Jw^2x/x_4 = -258/123 = -2.098$ (lbs. ins./microradian).

The computation for $F = 13{,}500$ C.P.M. is given in Table I. It is usual and convenient to give unit value to x_1, since this quantity is merely a multiplier throughout. Besides enabling the dynamic stiffness to be calculated, the table gives the " swinging form " or deformation shape in the x column, and also the transmitted torques in all the shafts in the column headed " ΣJw^2x." This data is of interest to engine designers, and in this respect the frequency tabulation method is superior to the other methods described. It should, however, be noted that it is quite possible to determine the swinging form and torque variation by the method of partial inertias (reference 5). It should also be noted that for convenience the factor 10^6 is dropped, since it is common to the value of w^2 and to the stiffnesses, but it must be remembered that the resulting value of Z is given in lbs. ins./microradian.

(§4) HEAVY SHAFTS.

When the torsional system includes one or more heavy shafts—*i.e.*, shafts whose polar moments of inertia cannot be regarded as negligible in comparison with the concentrated inertias—the only satisfactory method of determining the dynamic stiffness is by means of the transference formulæ (4.2) and (4.3), together with a similar formula for transference of the reference point through a heavy shaft. By normal mathematical analysis it can be shown (reference 6) that :—

$$CZ_2 = -\cot y + (\operatorname{cosec} y)/(\cos y - CZ_1 \sin y) \quad . \quad . \quad (4.4)$$

where :—

Z_1 and Z_2 are dynamic stiffnesses at either end of the shaft, Z_2 including the shaft, in lbs. ins./radian.

$I = wL\sqrt{(D/G)}$, $C = LGIy$, $w = 2\pi X$ (frequency in C.P.S.),

L = length of shaft in inches,

D = density of material of shaft in lbs. ins^{-4} secs.^{2},

G = shear modulus of material in lbs./in.^{2},

y = polar second moment of cross-sectional area in ins.^{4}.

The formula (4.4) refers only to uniform circular shafts, but subject to that condition can be used in place of (4.3) to transfer the reference point through the heavy shaft.

(§5) GEARING.

The presence of a gear-step in the system necessitates modification of the methods described above. The system may be treated in its original form by either the transference method or the frequency tabulation; alternatively the system can be " reduced " to a single shaft speed and then treated by either method.

(i) TRANSFERENCE METHOD.

Suppose that the dynamic stiffness Z_P of the system to the right of the point P in Fig. 5*a* is known; it is required to find the dynamic stiffness Z_Q at the point Q (*i.e.*, at the centre of the gear-wheel but excluding the wheel and all that lies to the left of it). Let the gear-ratio be $n : 1$; in the case of aircraft installations n is less than unity, as the crankshaft speed is greater than the propeller-shaft speed. Using subscripts P and Q to denote the reference points,

$$T_P = Z_P x_P, \quad T_Q = Z_Q x_Q,$$

and since

$$x_Q = n x_P, \text{ and } T_Q = T_P/n, \quad . \quad . \quad . \quad . \quad . \quad (4.5)$$

it follows that

$$Z_Q = T_Q/x_Q = Z_P/n^2 \quad . \quad . \quad . \quad . \quad (4.6)$$

Now suppose the system to be that shown in Fig. 5*b*, where the inertia J_4 of Fig. 3*a* has been replaced by a gear step, the gear ratio being 0.4 : 1. Using the notation of § 2, expressing all stiffnesses in lbs. ins./microradian, and taking a frequency of 13,500 C.P.M., from § 2

$$Z_P = 0.902, \text{ hence } Z_P' = Z_P - J_P w^2 = -0.098,$$
$$Z_W = Z_P'/n^2 = -0.098/0.16 = -0.6125,$$
$$Z_W' = Z_W - J_W w^2 = -3.6125.$$

The dynamic stiffness Z_R at R, the remote end of the propeller shaft, is given by (4.3) as

$$Z_R = Z_W' K_R/(Z_W' + K_R) = 0.2117.$$

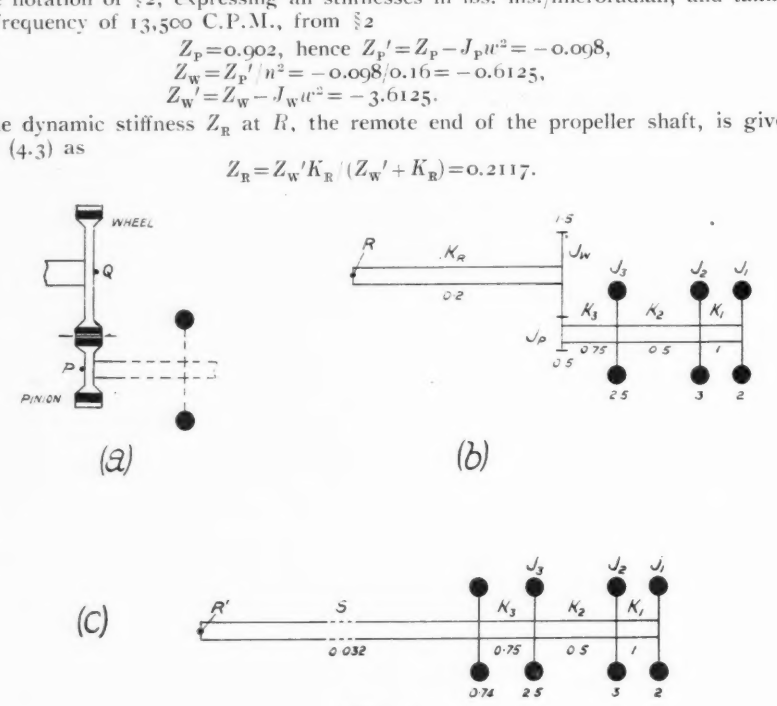


FIG. 5.

(ii) FREQUENCY TABULATION METHOD.

The relations (4.5) can be utilised in the frequency tabulation method. The computation is given in Table II for $F = 13{,}500$ C.P.M. The first three rows, and the x-value in the fourth row, are the same as in Table I. With $J_P = 0.5$,

TABLE II.

FREQUENCY TABULATION FOR SYSTEM OF FIG. 5*b*.

$w^2 = 2 \times 10^6$.

Inertia ref.	J	Jw^2	x	Jw^2x	ΣJw^2x	K	d
1	2	4	1	4	4	1	4
2	3	6	−3	−18	−14	0.5	−28
3	2.5	5	25	125	111	0.75	148
P	0.5	1	−123	−123	−12	—	—
—	—	—	−49.2	—	−30	—	—
W	1.5	3	−49.2	−147.6	−177.6	0.2	−888
—	—	—	838.8				

$$Z_R = -(-177.6)/838.8 = 0.2117 \text{ (lbs. ins./microradian).}$$

$J_P w^2 = 1.0$, and the fifth and sixth columns of that row can be filled in. The entry -12 is the total torque at the pinion, and this figure is divided by n ($=0.4$) to give the equivalent torque at the wheel, while the amplitude value $x_P = -123$ is multiplied by n to give the amplitude at the wheel (-49.2). The table can therefore be completed, and it will be seen that the result agrees with that obtained in (i) above.

(iii) Reduction to Single Reference Shaft.

To reduce the system to a single reference shaft, the inertias and shaft stiffnesses to one side of the gear are converted to equivalent inertias and stiffnesses with respect to the other part of the system. The method of conversion is established as follows: Since all parts of the system vibrate with the same frequency, and inertias have the dimensions of (stiffness/w^2), the conversion rule for inertias and shaft stiffnesses will be the same as that for dynamic stiffnesses. From (4.6), $Z_P = n^2 Z_Q$, and the inertia of the wheel in Fig. 5*b* referred to the engine-shaft is $1.5 \times 0.4^2 = 0.24$; similarly the stiffness of the propeller shaft similarly referred is $0.2 \times 0.4^2 = 0.032$. The system can therefore be reduced as shown in Fig. 5*c*; the gears are replaced by a single inertia $0.74 = 0.5 + 0.24$, and the shaft S has a stiffness value 0.032. Table III shows the modification to Table II, commencing at the fourth line. The dynamic stiffness at R' referred the engine shaft is 0.0339, and referred to the propeller shaft $Z_R = Z_R'/0.4^2 = 0.2117$ as before. It must be noted that the entries in the x-column must be multiplied by 0.4 in order to obtain the true amplitudes at the wheel and at R.

Table III.

Modification to Table II for Reduced System.

J	Jw^2	x	Jw^2x	ΣJw^2x	K	d
0.74	1.48	-123	-182.04	-71.04	0.032	-2220
—	—	2097				

$$Z_R' = -(-71.04)/2197 = 0.0339$$
$$Z_R = 0.0339/0.4^2 = 0.2117$$

Note.

It will be evident from numerical work that a system, of the type considered in this analysis, having n degrees of vibrational freedom (*i.e.*, which, when reduced to a single reference shaft, has $n+1$ inertias joined by n shafts) has its dynamic stiffness zero for n distinct frequencies, so that the system has n distinct natural frequencies. This fact is normally demonstrated, with mathematical rigour, by a consideration of the determinant formed from Langrangian equations of motion; a simple proof is easily derived, however, by means of the concept of dynamic stiffness, and this proof is given below in the Appendix.

(§6) Torsional Dynamic Stiffness of Propellers.

The torsional dynamic stiffness of a propeller is best obtained experimentally as described in part (*a*) of this section. There are methods of calculating the function, but such a procedure is tedious and complicated, although the results are in surprisingly good agreement with those of practical tests. Owing to the stiffening effect of centrifugal force an allowance for rotation must in any case be made; details of the method by which this is done are available to manufacturers and other organisations concerned with the problem.

The shape of the dynamic stiffness curve for a propeller is also complicated by the fact that the blades are capable of vibrating both in the plane of rotation and normal to this plane, and that these two motions are coupled through the twist in the blades and also through the pitch setting. If there were no twist in the blades, so that the major axes of all blade sections made the same angle

with the plane of rotation, a sinusoidal torque imposed at the hub would excite flatwise and edgewise vibrations whose relative amplitudes could be calculated by resolving the torque into flatwise and edgewise bending moments. Owing to the twist, however, the terms " flatwise " and " edgewise " lose their meaning. Possible resonant conditions are best predicted by obtaining the dynamic stiffness curves for the non-rotating propeller by experimental means and then correcting for centrifugal force by calculation; a strain-gauge test of the propeller when operating on the engine continues to be the only satisfactory means of determining blade stresses.

V. Tuning Inertias; Simple Typical Systems.

Suppose that the effective inertia of a system at a point P is J_e for a frequency F. If a system whose effective inertia at this frequency is $-J_e$ be attached at P, the total effective inertia at P is then zero, so that F is a natural frequency of the combined system. The auxiliary attached system is termed a " tuning system," and the inertia $-J_e$ is called the " tuning inertia " of the original system for the frequency F. Thus if it is desired to tune a system to have F as a natural frequency it is only necessary to determine at this frequency the effective inertia J_e at any point P, and to attach at P a tuning system whose effective inertia at P for the frequency F is $-J_e$. Denoting the tuning inertia by J_T, there is the relation

$$J_T = -J_e \qquad (5.1)$$

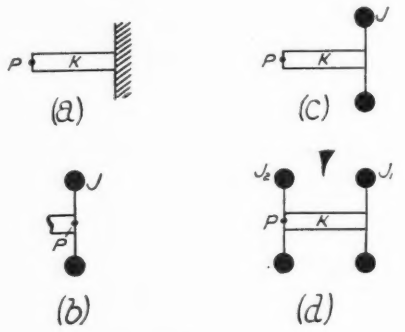


Fig. 6.

It is to be noted that if the effective inertia of the main system is negative, so that the tuning inertia is positive, tuning can be effected by the rigid attachment of a single inertia. Thus, if it is desired to operate the engine in Fig. 5*b* for a workshop test with 13,500 C.P.M. as a natural frequency, a flywheel should be rigidly attached at R whose inertia is 0.1059 lbs. ins. secs.^{2}, since this is the tuning inertia corresponding to a dynamic stiffness value 0.2117 lbs. ins./radian, from equations (1.3) and (5.1). If, on the other hand, the effective inertia is positive, so that the tuning inertia is negative, it is not possible to tune the system by the rigid attachment of a single flywheel, and some other tuning system must be used. The dynamic stiffness at one end of a light shaft, the other end of which is constrained to have zero torsional displacement, is simply the static stiffness, so that the effective inertia is negative by reason of (1.3). Thus it might be possible to make use of some variety of torsional spring, but it is doubtful whether the suggestion has any practical importance, since it is usually necessary for torsional systems to be capable of steady rotation. The more usual method employed is the attachment of a shaft driving an inertia.

The effective inertias of various simple tuning systems are given in Table IV, the systems being illustrated in Fig. 6. The last two columns in the table give the frequency ranges for which the effective inertias are positive and negative respectively; in these ranges the systems can be used to tune main systems having negative and positive effective inertias respectively.

TABLE IV.

TUNING RANGES FOR SIMPLE TUNING SYSTEMS.

System (see Fig. 6)	J_e	Frequency Ranges. $J_e > 0$ (to tune main system in which $J_e < 0$).	$J_e < 0$. (to tune main system which $J_e > 0$).
a	$-K/w^2$	—	all
b	J	all	—
c	$KJ/(K-Jw^2)$	$w^2 < K/J$.	$w^2 > K/J$
d	$\dfrac{K(J_1+J_2)-J_1J_2w^2}{K-J_1w^2}$	$\begin{cases} w^2 < K/J_1 \text{ and} \\ w^2 > K(J_1+J_2)/J_1J_2 \end{cases}$	$\begin{cases} K/J_1 < w^2 \\ < K(J_1+J_2)/J_1J_2 \end{cases}$

Type (*c*) requires some comment. Let $w_n^2 = K/J$, and further let $X = w/w_n$ = (frequency of vibration)/(natural frequency of tuning system). Then the expression for J_e can be put in the form

$$J_e = J/(1-X^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.2)$$

The form of the curve of J_e plotted against X is as shown in Fig. 7. Any negative value of J_e can be obtained, but only those positive values greater than J are available.

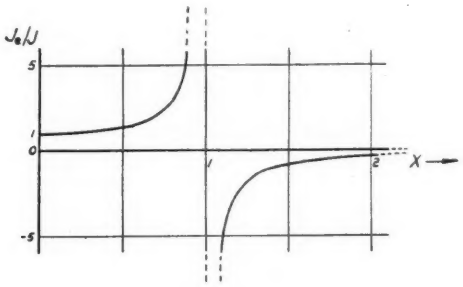


FIG. 7.

A very complete account of the theory and practice of tuning torsional systems is given by Ker Wilson (reference 7).

VI. SOME APPLICATIONS OF THE METHOD.

Some applications of the effective inertia method of analysis are discussed briefly in this section.

(*a*) AIRCRAFT POWER-PLANT WITHOUT DYNAMIC ABSORBERS.

The dynamic stiffness of the propeller with reference to the hub can be determined experimentally as described in section IV*a*, and the dynamic stiffness of the engine can be calculated by any of the methods of section IV*b*. The stiffening effects of centrifugal force can be allowed for by calculation.

Torsional resonant frequencies are obtained by the method of intersections (see section III).

(*b*) Effect of Propeller Pitch Angle.

The effects of varying the pitch setting of propeller blades can be determined quantitatively by numerical methods; qualitative results are more easily obtained, however, from consideration of an idealised propeller having prismatic blades of rectangular cross-section. An analytic expression can then be obtained for the torsional dynamic stiffness of the blades at the hub, and this expression can be represented by graphs drawn for different pitch-settings.

(*c*) Pendulum-Type Absorbers.

The effective inertia of a pendulum-type torsional vibration absorber can be determined mathematically. Zdanowitch and Wilson (reference 8) give expressions for the effective inertia of the most commonly used types of absorber, including the types most favoured by aero-engine designers, and Ker Wilson (reference 9) gives a full description of the methods of application.

Most authorities are now agreed that the concept of effective inertia provides the best method of dealing with absorber problems. The torsional vibration characteristics of a system including pendulum absorbers are easily derived from the frequency tabulation (section IV*b*, iii) extended so as to allow for the effective inertia of the absorbers at their respective crank-throws.

(*d*) Coupling of Torsional and Axial Modes Due to Helical Gearing.

A good example of the way in which the method of dynamic stiffness can eliminate much work in analysis is afforded by the solution to the following problem:—

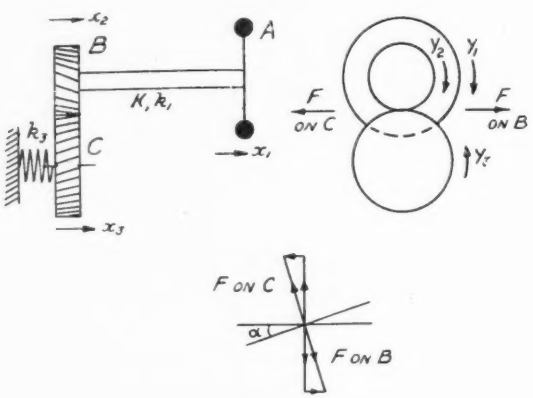


Fig. 8.

In Fig. 8, *A* is a turbine, driving one half of a hydraulic coupling *C* via the helical gear-pinion *B* and the gear-wheel machined on the coupling *C*; it is required to find the natural frequencies of the system.

The following assumptions are made:—

(i) The gears are without backlash;

(ii) Torsional stiffness is confined to the shaft between turbine and pinion;

(iii) There is no axial restraint at the turbine thrust-bearing (a reasonable assumption for small displacements taken up by bearing clearances); and

(iv) The flexibility of the wheel thrust-bearing provides spring control of axial motion.

With the notation :—

Part.		Mass.	Polar Inertia.	Displacements. Axial.	Torsional.	Radius.
Turbine	A	m_1	J_1	x_1	Y_1	—
Pinion	B	m_2	J_2	x_2	Y_2	r
Wheel	C	m_3	J_3	x_3	Y_3	R

K = torsional stiffness of shaft AB,
k_1 = axial stiffness of shaft AB,
k_3 = axial stiffness at wheel C,
w = phase-velocity of vibration,
α = helix-angle,
F = gear-tooth reaction,
Z_{T2} = torsional dynamic stiffness of system AB referred to B,
Z_{T3} = torsional dynamic stiffness of wheel C referred to C,
Z_{A2} = axial dynamic stiffness of system AB referred to B,
Z_{A3} = axial dynamic stiffness of wheel C referred to C,

the geometric condition is

$$(x_2 - x_3)\tan\alpha = rY_2 - RY_3$$

and

$$\left.\begin{aligned} -Fr\cos\alpha &= Z_{T2}Y_2 \\ FR\cos\alpha &= Z_{T3}Y_3 \\ F\sin\alpha &= Z_{A2}x_2 \\ -F\sin\alpha &= Z_{A3}x_3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6.1)$$

Substitution for x and Y from the last four equations of (6.1) in the first equation gives

$$(1/Z_{A2} + 1/Z_{A3})\sin^2\alpha + (r^2/Z_{T2} + R^2/Z_{T3})\cos^2\alpha = 0.$$

Expressing the dynamic stiffnesses in terms of the constants of the systems this last equation gives :—

$$\left[\frac{m_1w^2 - k_1}{k_1(m_1+m_2) - m_1m_2w^2} + \frac{w^2}{k_3 - m_3w^2}\right]\sin^2\alpha + \left[\frac{r^2(J_1w^2 - K)}{K(J_1+J_2) - J_1J_2w^2} - \frac{R^2}{J_3}\right]\cos^2\alpha = 0 \quad . \quad . \quad (6.2)$$

In order to derive the frequency equation (6.2) by normal mathematical methods from the force and torque equations of the system it is necessary to reduce a seventh order determinant in x_1, x_2, x_3, Y_1, Y_2, Y_3, F. The saving in labour effected by the use of the dynamic stiffness method will be appreciated by anyone who has had occasion to manipulate high-order determinantal equations with literal elements.

If the various phase-velocities associated with the natural frequencies of the system are as follows :—

w_T for torsional motion when $\alpha = 0$,
w_t for torsional motion of turbine/pinion system when $\alpha = 90°$,
w_{A1}, w_{A2} for axial motion when $\alpha = 90°$,
w_{a1}, w_{a2} for axial motion of turbine/pinion system and of wheel when $\alpha = 0$,

then (6.2) can be put in the convenient form :—

$$(1/m_2 + 1/m_3)(w_{A1}^2 - w^2)(w_{A2}^2 - w^2)(w_t^2 - w^2)\sin^2\alpha + (r^2/J_2 + R^2/J_3)(w_{a1}^2 - w^2)(w_{a2}^2 - w^2)(w_T^2 - w^2)\cos^2\alpha = 0 \quad . \quad (6.3)$$

from which it is easy to deduce special cases of interest.

(*e*) " BRANCHED " SYSTEMS.

The " branched " type of torsional system, such as is found in modern in-line aero-engines with twin crankshafts, can be treated adequately by the transference method. The characteristics of various parts of the system are easily determined by any of the calculation methods, and use of the transference formulæ enables

the dynamic stiffness at any point of the complete system to be found. It does not seem necessary or desirable to give any example of this application.

Conclusions.

The concepts of effective inertia and dynamic stiffness, the application of which enables the vibration characteristics of torsional systems to be investigated both numerically and analytically, are among the most important of modern developments in vibration theory. Certainly the methods of study based on these concepts have been adopted as " routine " methods by engine and propeller manufacturers in this country, and in other applications neat and comparatively trouble-free solutions can be formulated by these means.

The choice between effective inertia and dynamic stiffness for a basis of analysis must depend upon the particular characteristics of the problem under consideration. In certain cases it may be convenient to use both quantities; for example, in the determination of the resonant frequencies in aircraft power plants equipped with pendulum-type absorbers, the effective inertias of the absorbers are utilised in the frequency tabulation to obtain the dynamic stiffness of the engine.

Linear (spring-plus-mass) and torsional systems are readily treated by these methods; an extension of the theory to cover fully the bending of beams in an equally simple manner would be very welcome.

Appendix.

A proof of the theorem that a " straight chain " torsional system with n inertias connected by $n-1$ shafts has $n-1$ distinct natural frequencies.

By a " straight chain " system is meant a system such as that of Fig. 3*a* or Fig. 5*c*, in which inertias and shafts alternate in a chain. Let the dynamic stiffnesses of the systems to the right of two points which are themselves immediately to the left of the nth and $(n+1)$th inertias be Z_n, Z_{n+1} as shown in Fig. 9. The transference formulæ (4.2) and (4.3) give

$$Z_p = \frac{K_n Z_n}{(K_n + Z_n)} - J_p w^2 \quad . \quad . \quad . \quad . \quad . \quad (A.1)$$

where for convenience p has been written for $(n+1)$. Now $Z_1 = -J_1 w^2$, and hence when $w^2 = 0$, $(Z_1)_0 = 0$; similarly, when $w^2 = \infty$, $(Z_1)_\infty = -\infty$. But from (A.1), $(Z_p)_0 = 0$ if $(Z_n)_0 = 0$, and hence by induction

$$(Z_p)_0 = 0 \quad . \quad . \quad . \quad . \quad . \quad (A.2)$$

Similarly $(Z_p)_\infty = -\infty$ if $(Z_n)_\infty \neq -K_n$; and by induction

$$(Z_p)_\infty = -\infty \quad . \quad . \quad . \quad . \quad . \quad (A.3)$$

Furthermore, let $D_n = d/d\,(w^2)\,(Z_n)$, then (A.1) gives

$$D_p = \frac{K_n^2 D_n}{(Z_n + K_n)^2} - J_p$$

so that if D_n is negative so also is D_p. But D_1 is negative, hence

$$D_p \leqslant 0 \quad . \quad . \quad . \quad . \quad . \quad (A.4)$$

Finally, if

$$Z_n = -(K_n - 0), \quad Z_p = -\infty,$$

and if

$$Z_n = -(K_n + 0), \quad Z_p = \infty,$$

so that if $Z_n = -K_n$ for N distinct values of w, Z_p has N infinite discontinuities. Thus from (A.2, 3, 4), if Z_n has N full negative ranges (*i.e.*, if there are N distinct values of w which give Z_n any specified negative value) then Z_p has $N+1$ full negative ranges; but Z_1 has one full negative range, and hence Z_n has n full negative ranges and $n-1$ infinite discontinuities; there are therefore $n-1$ finite (*i.e.*, non-zero) values of w for which Z_n is zero.

It has thus been established that a straight chain torsional system with n inertias connected by $n-1$ shafts has $n-1$ distinct natural frequencies.

ACKNOWLEDGMENTS AND BIBLIOGRAPHY.

The author is indebted to the de Havilland Aircraft Company for permission to publish this paper, and wishes to thank Miss M. K. B. Harwood and Miss J. A. R. Howlett for their help in the preparation of the manuscript.

REFERENCES.

(The initials " F.V.S." refer to the author's textbook: " Fundamentals of Vibration Study," Chapman & Hall.)

1. Author. " Mechanical Impedance of Damped Vibrating Systems," Journal of the Royal Aeronautical Society, November, 1941.
2. F.V.S., p. 61.
3. B. C. Carter. " Harmonic Torque Exciters," Aircraft Engineering, August, 1939.
4. F.V.S., p. 54.
5. F.V.S., p. 56.
6. F.V.S., p. 72.
7. Ker Wilson. " Practical Solution of Torsional Vibration Problems," Champan & Hall.
8. R. W. Zdanowitch and T. S. Wilson. " Elements of Pendulum Dampers," Proceedings of the Institution of Mechanical Engineers, June, 1940.
9. Ker Wilson. Op. cit.
10. M. A. Biot. " Coupled Oscillations of Aircraft Engine Propeller Systems," Journal of the Aeronautical Sciences, July, 1940.
11. Author. " The Electro-Mechanical Analogy in Oscillation Theory," Journal of the Royal Aeronautical Society, January, 1943.
12. Ker Wilson. Op. cit.

" THE ELECTRO-MECHANICAL ANALOGY IN OSCILLATION THEORY."

By R. G. Manley (Student Member).

Introduction.

It has long been realised that there is a precise analogy between the properties of vibrating mechanical systems and those of alternating current electric circuits. It was from this analogy that the concept of " mechanical impedance " (more appropriately termed " dynamic stiffness ") was derived, thus enabling the theory of certain types of mechanical vibration, notably torsional motion, to be put on a more rational basis (see references 1 and 2).

By means of analogous elements it is possible to represent a mechanical vibrating system, provided it includes no bending elements, by an equivalent electric circuit. The author has been unable to discover any full description of the process, and the object of the present paper is to set out the principles underlying the procedure and to illustrate them briefly by examples.

1. Analogous Elements.

(i) Inertia—Inductance.

The acceleration $\ddot{x}$ produced in a mass by the application of a force p in the direction of x is given by

$$p = m\ddot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

and in the torsional case the angular acceleration $\ddot{Y}$ produced in an inertia J by the action of a torque T is given by

$$T = J\ddot{Y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

The electrical equivalent to force is e.m.f., and the quantity analogous to mass or inertia will be that quantity which, when multiplied by the second time-differential of some other quantity, gives an e.m.f. Inductance has the units of e.m.f. divided by the rate of change of current, and current itself is the rate of change of electrical quantity (q); in fact, the quantity q of electricity flowing through an inductance L is related to the potential difference e across the inductance by the equation

$$e = L\ddot{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1c)$$

Thus if e.m.f. is taken to be analogous to force, and electric quantity (q) to linear displacement, inductance is analogous to mass; while in the torsional case inductance is analogous to inertia.

(ii) Flexibility—Capacity.

The displacement x between the ends of a spring of flexibility f is related to the force p transmitted by the spring by the equation

$$p = x/f \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

and if torsional flexibility is represented by F, the equivalent expression for torsional loading is

$$T = Y/F \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

In order to obtain a consistent analogy, the electrical property analogous to flexibility must have the dimensions of electrical quantity divided by e.m.f.; this requirement is filled by capacity (C):—

$$e = q/C \quad . \quad . \quad . \quad . \quad . \quad . \quad (2c)$$

(iii) DAMPING—RESISTANCE.

The relative velocity $\dot{x}$ between the ends of a "dashpot," whose damping coefficient is b (see reference 3), produced by a transmitted force p is given by

$$p = b\dot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

while if the torsional damping coefficient is B the equivalent expression for torque is

$$T = B\dot{Y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

Since current is the rate of change, or time-differential, of electrical quantity, the electric property analogous to damping must have the dimensions of e.m.f. divided by current, *i.e.*, resistance (R):—

$$e = Ri = R\dot{q} \quad . \quad . \quad . \quad . \quad . \quad (3c)$$

where i is the current.

The analogous quantities are set forth in the table.

TABLE OF ANALOGOUS QUANTITIES.

Mechanical.				Electrical.	
Linear.		Torsional.			
displacement	x	displacement	Y	quantity	q
velocity	$\dot{x}$	velocity	$\dot{Y}$	current	$\dot{q}$
force	p	torque	T	e.m.f.	e
mass	m	inertia	J	inductance	L
flexibility	f	flexibility	F	capacity	C
damping coeff.	b	damping coeff.	B	resistance	R

II. CONVERSION OF SYSTEMS—PRINCIPLES.

Springs and dashpots have, in general, two associated displacements, namely, the displacement at each end, and the force transmitted by them is proportional to the difference between these two displacements in the case of springs, and to the rate of change of this difference in the case of dashpots. Translating into the equivalent electrical terms, capacities and resistances have, in general, two currents flowing through them in opposite directions, so that the potential difference across a capacity is proportional to the time-integral of the difference between the two currents, while the potential difference across a resistance is proportional to this difference. The capacities and resistances which represent springs and dashpots must therefore, in general, be included in two circuits; but since a mass or inertia has only one displacement, the inductance representing it is included in only one circuit. These points are illustrated in Fig. 1.

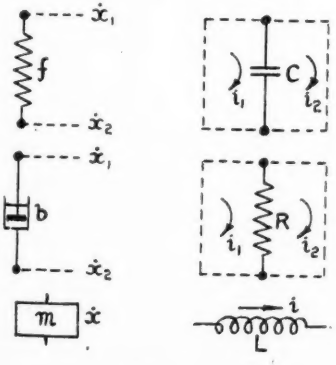


FIG. 1.

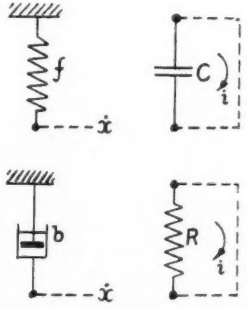


FIG. 2.

If one end of a spring or dashpot is fixed, one of the associated electric circuits becomes open and can be omitted (see Fig. 2).

A spring-dashpot parallel assembly such as that shown in Fig. 3*a* is represented by a series assembly of capacity and resistance, as shown in Fig. 3*b*, since the capacity and resistance must carry the same current (the displacements of the spring and dashpot being necessarily equal). Similarly, if a force is applied to a mass (Fig. 4*a*) the electrical equivalent is as shown in Fig. 4*b*.

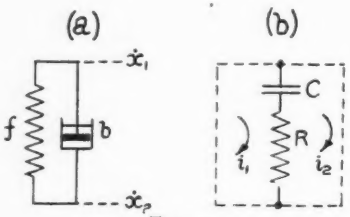


FIG. 3.

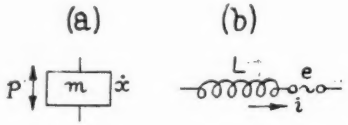


FIG. 4.

It is perhaps unfortunate that the same symbol is used in diagrams to represent springs and resistances, which are not analogous elements; but with due attention to the rest of the system there should be no confusion.

III. Conversion of Systems—Examples.

The single-mass damped system shown in Fig. 5 is equivalent to a combination of the circuits of Fig. 3, in which $i_1 = 0$, and Fig. 4. Regarded in another way, the electric elements representing the mass, spring, dashpot and applied force must all be in series, so that they carry the same current, for the mechanical elements all have the same displacement, and hence the same velocity.

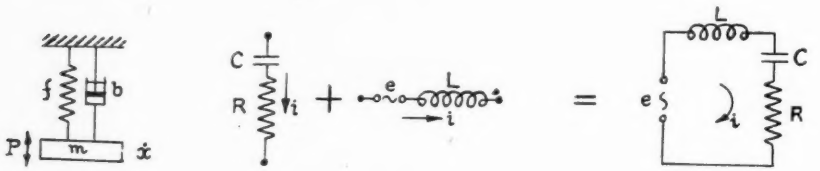


FIG. 5.

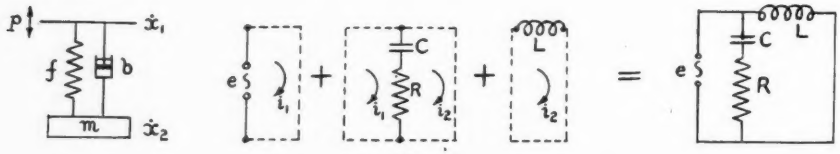


FIG. 6.

The system shown in Fig. 6 (the simple linear seismic vibrograph) incorporates two displacements as shown. The electrical circuit is most readily built-up in stages: (i) The input stage consisting of the e.m.f. "e" with a dummy circuit carrying a current i_1; (ii) the coupling stage, consisting of a capacity and resistance series-connected and carrying two currents i_1 and i_2 in opposite directions; and (iii) the load stage, which consists of an inductance carrying a current i_2. The manner in which the partial circuits are combined to form the complete circuit is easily seen from the diagram.

A variation of this system is depicted in Fig. 7, the main mass (1) being connected to the moving top support both by the spring (2) and by the dashpot-mass-spring chain (3, 4, 5).

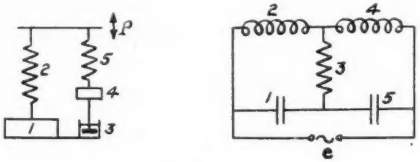


FIG. 7.

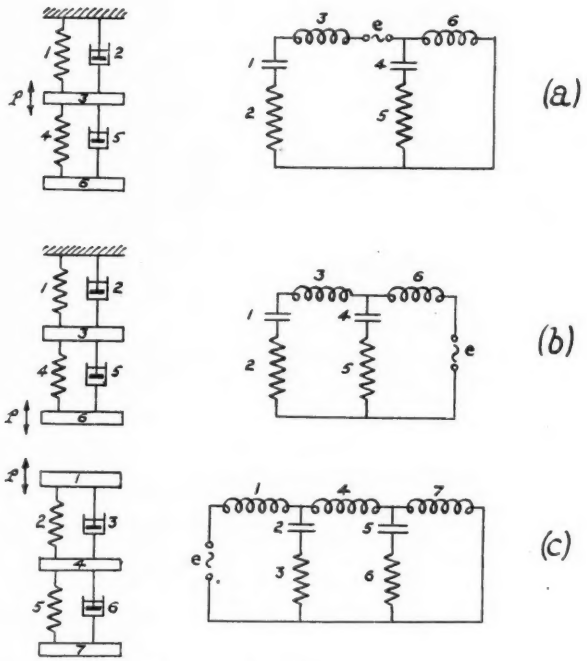


FIG. 8.

Some further examples of equivalent circuits are given in Figs. 8 and 9. In Fig. 9 transformers are introduced as the analogues of torsional gear-steps, but it must be noted that the analogy is not quite precise; the ratio of the torques in the two shafts connected to the gears is dependent on the gear-ratio, not on the ratio of the inertias of the gears, as the electric circuit would seem to suggest. So long as the gear inertias are small compared with the other inertias in the system, however, there will be little quantitative error.

The system in Fig. 9*d* represents the case of torsional vibration excited by the propeller in an engine/propeller system (it is assumed that the propeller is effectively a rigid flywheel; this assumption is not justified in practice, but for moderate frequencies the condition is approached in the case of wooden propellers).

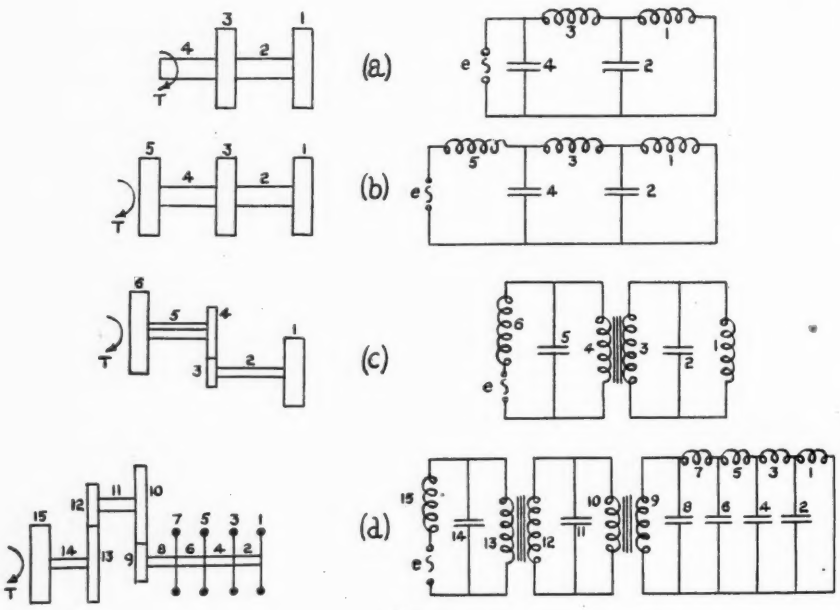


FIG. 9.

ACKNOWLEDGMENTS AND REFERENCES.

The author is indebted to the de Havilland Aircraft Company for permission to publish this paper, and desires to thank Mr. F. Campbell Rose, by whom his attention was first drawn to the subject.

REFERENCES.

1. Author. "The Mechanical Impedance of Damped Vibrating Systems," Journal of the Royal Aeronautical Society, November, 1941.
2. Author. "Dynamic Stiffness and Effective Inertia Methods in Modern Torsional Vibration Theory," Journal of the Royal Aeronautical Society, January, 1943.
3. Author. "Fundamentals of Vibration Study," Chapman & Hall, p. 17.

REVIEWS.

Aircraft Radio.

By D. Hay Surgeoner. (2nd Ed.) Sir Isaac Pitman and Sons. 1942. 15/-.

In the four years that have elapsed since this book was first published considerable progress has been made, especially in night flying equipment and landing approach installations, and the chapters dealing with these subjects have been revised accordingly. The information in Chapter V has been made much clearer by the addition of appropriate sub-titles, and now includes the series of diagrams upon the Radio Compass and its uses which was formerly included in Chapter VII. The notes on automatic radio control of aircraft have been entirely rewritten. The next chapter now contains some useful notes on the Beam Approach and use of the Glide Path when landing, with a diagram illustrating the general layout of the Beam Approach ground equipment. The chapter on Aircraft Equipment and its Operation has been very largely rewritten and has been improved by rearrangement. Under " Airport Equipment " we note that what was formerly known as the Lorenz system is now described as the " Standard " Beam Approach System. The final chapter, dealing with Airport and Airway Lighting, is substantially unaltered.

Air Navigation.

By E. R. Hamilton. Thos. Nelson and Sons. 1942. 5/-.

By the nature of the subject, books on Air Navigation must be as alike as peas in a pod, since there are a limited number of basic problems and certain standard methods of dealing with them. On close inspection, however, some peas have maggots in them and others have not. We can recommend this particular pea as thoroughly sound. The author has been teaching Navigation since the last war and approaches the subject in a practical spirit. An excellent idea is the provision of a separate practice chart, in a pocket on the back cover, replacement copies of which can be bought when needed. The hints given on plotting (pp. 83-85) are very useful and should go a long way towards helping the student to turn out neat and shipshape work. In most books of this type the hypnotic effect of the 2nd Class Navigator's examination syllabus leads to the forcible insertion of some perfunctory notes on Meteorology. Mr. Hamilton has escaped this pitfall and has used the space saved by this omission to devote two final chapters to the elements of Astro-navigation and Radio-navigation. Exercises are given at frequent intervals, with answers at the end of the book. The standard is slightly higher than that required for the A.T.C., but the style is so clear that even a complete beginner could work through the book without being mystified.

Victory Through Air Power.

Major Alexander Seversky. Hutchinson and Co. Ptd. 1942. 9/6 net.

This is a book to be read by (1) all those who go to sea in ships (2) all those who are concerned with fighting on the land (3) the ordinary man in the street, and (4) all those in aviation, and in that order. It will do the first two a lot of good, exasperate them, and it is hoped encourage them to believe that any naval or military operation is at least more likely to succeed with air support than without it. The ordinary man in the street will want less convincing and all those in aviation have preached yesterday, are preaching to-day, and will continue to preach to-morrow exactly what Major Seversky states in this book at length.

Major Seversky has had a long experience in aviation and has become well aware of the power which aviation can wield. His book is written from the American point of view, is written with the propagandist point of view of demanding a separate Air Force in the U.S.A. But this is by the way. He is only trying to do for America what has been done in this country, the provision of a separate air force which is governed by those who think in air terms, and not in terms of naval and military strategy.

His very first sentence is a challenge and is the thesis of his book. "The most significant single fact about the war now in progress is the emergence of aviation as the paramount and decisive factor in war-making."

The whole book, indeed, is a challenge to all those who have not become acutely aware that the stone age, the bronze age and the iron age have gone for ever, that henceforward the people of the earth must look upwards for their salvation.

Here is chapter and verse of the power of the air. Because that power is still being used inefficiently there are still people who can point to the mistakes and say that sea power is vital for the Allied Nations, that large navies are necessary to win us the war. How many lives there are yet to be lost due to such thinking one dreads to think about.

Many of Major Seversky's arguments and conclusions are based on the wrong premises, but most of his conclusions are right. This is a book to be read and argued about, for it is worth both.

WHAT'S THE GEN?

By H. W. J. Crowther, Ltd. 1942. 1/3 net.

SSH! GREMLINS.

By H. W. J. Crowther, Ltd. 1942. 2/- net.

Two small books for amusement only, and two small books which will amuse.

What's the Gen? is an illustrated list of R.A.F. slang, and here will be found most of these terms which are peculiar to squadrons all over the world, words which are slang to-day but many of which will be in the Oxford Dictionary in a few years time as part of the language of the nation. The illustrations by Stacey add to the amusement.

Gremlins are to be seen to be believed and Neb's illustrations make one wonder under what conditions they are seen. The author says Gremlins like to be taken seriously, so the author had better be careful. By the way the author of both these books is the same man who wrote that delightful book "Air Over Eden."

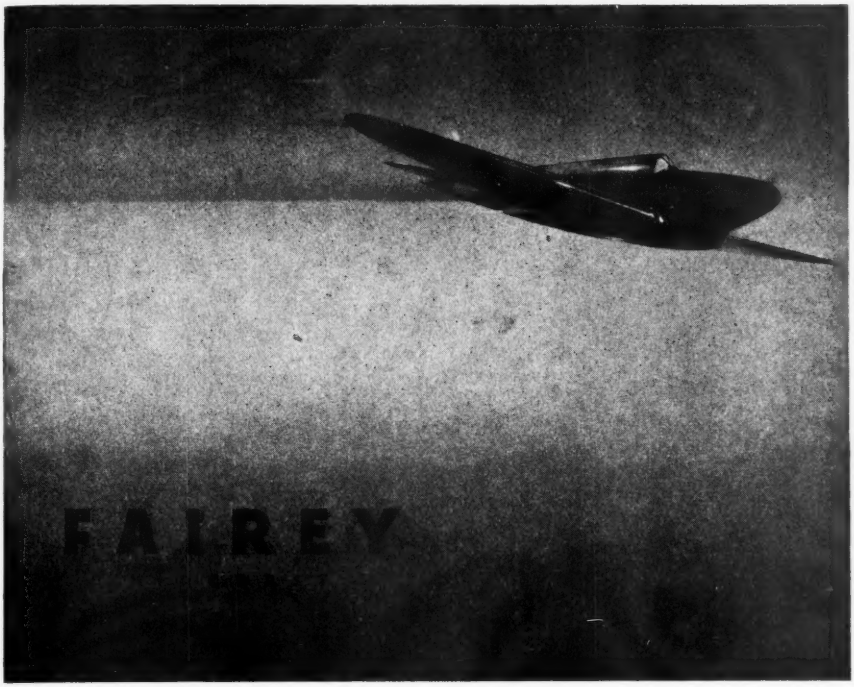
PILOTS OF FIGHTER COMMAND.

Sixty-four portraits by Captain Cuthbert Orde, with a foreword by Air Chief Marshal Sir Sholto Douglas, K.C.B., M.C., D.F.C., and an introduction by Air Vice-Marshal J. C. Slessor, C.B., D.S.O., M.C. G. Harrap and Co., Ltd. 1942. 25/- net.

Air Vice-Marshal Slessor, in his introduction, says: "If it be true, as it obviously must be, that an intimate knowledge and a close insight into the character of the sitter are an enormous advantage to an artist, then both Fighter Command and Captain Orde are fortunate."

And it will be a fortunate recipient who is given this book. Captain Orde spent the best part of a year on almost every operational station of the Command and the results must be a lasting memento.

Captain Orde adds to his great talents as an artist a gift for writing which he has put to good use in his own introduction and description of the fighters he has portrayed. A book to buy, to keep, and to enjoy.



Pepys into the future...

YOU cannot write history before it happens. Few, for example, would presume to enter into their diary the date and hour of victory.

In aviation, the only thing we know with certainty about the future is that there will be, inevitably, tremendous and far-reaching developments in civil flying. A vast air-minded public, their former prejudices swept away, will be eager to travel by air.

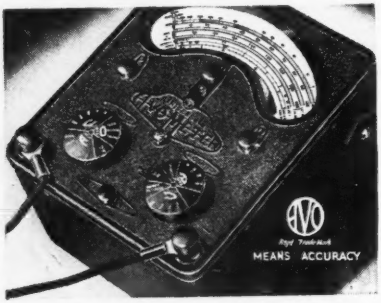
Wings are indeed the wheels of tomorrow and, although details of shape, speed and design are hidden from us, General Aircraft Limited, with their wide experience and research into the simplification of flying, are well equipped to play their part in fashioning those wings.

WINGS ARE THE WHEELS OF TO-MORROW

GENERAL AIRCRAFT LIMITED *Enquiries to:*– Sales Dept., London Air Park, Feltham, Middlesex.

Overseas enquiries to our Export Division:– ENGINEERING & AVIATION EXPORTS LIMITED, 50, PALL MALL, LONDON, S.W.1. Telephone: Abbey 6210.

H 75



THE MODEL 7 *46-Range Universal*

AVOMETER

Electrical Measuring Instrument

A self-contained, precision moving-coil instrument, conforming to B.S. 1st Grade accuracy requirements. Has 46 ranges, providing for measuring A.C. & D.C. volts, A.C. & D.C. amperes, resistance, capacity, audio-frequency power output and decibels. Direct readings. No external shunts or series resistances. Provided with automatic compensation for errors arising from variations in temperature, and is protected by an automatic cut-out against damage through overload.

Some delay in delivery of Trade orders is inevitable, but we shall continue to do our best to fulfil your requirements as promptly as possible.

Sole Proprietors and Manufacturers:

AUTOMATIC COIL WINDER & ELECTRICAL EQUIPMENT CO., LTD.

Winder House, Douglas St, London, S.W.1. *Vic.* 3404/7

JOURNAL OF THE AERONAUTICAL SCIENCES

Foreign Subscriptions:

1 year $10.00; single copies $1.00

AERONAUTICAL ENGINEERING REVIEW

Foreign Subscriptions:

1 year $3.00; single copies $0.30

Combined Subscription Rate:

1 year $12.50

Published Monthly

by the

INSTITUTE OF THE AERONAUTICAL SCIENCES

1505 RCA Building West, 30 Rockefeller Plaza, NEW YORK CITY

Information on membership in the Institute will be sent on request.

AIR NAVIGATORS

Makers of all types of Protractors for nearly thirty years, we are producing the celebrated "DOUGLAS" Protractor in ever increasing quantities. The "DOUGLAS" Protractor is indispensable to all navigators, supplied in two sizes, 5" and 10" square.

The "MACLEAN" D.R. (Dead Reckoner) Protractor of new design is now in full production. Designed primarily for Air Navigation, it is simple to use and serves as a constant reminder of essential data.

For further particulars of these two instruments and all other navigational aids, write for our illustrated brochure to

LONDON NAME PLATE
MANUFACTURING CO. LTD
ZYLO WORKS BRIGHTON 7

Also won by a T.I.G.B. Student

MAY, 1942, A.F.R.Ae.S.
BADEN-POWELL PRIZE

Qualify by HOME-STUDY with The T.I.G.B.

T.I.G.B. Students have now gained

25 FIRST PLACES

in the Royal Aeronautical Society's Examination (A.F.R.Ae.S.)

Write to-day for "The Engineer's Guide To Success"—Free—containing the world's widest choice of engineering and aeronautical courses—over 200—covering all branches and which alone gives the Regulations for Qualifications such as **A.F.R.Ae.S., Ground Engineers, Air Navigators, A.M.I.Mech.E., A.M.I.E.E., C. & G., etc.** The T.I.G.B. guarantees training until successful.

The Technological Institute of Great Britain
39 Temple Bar House, London, E.C.4

Founded 1917
20,000 Successes

World's best choice of Courses in Engineering

THE ME PATENT

UNIVERSAL BALL JOINTS

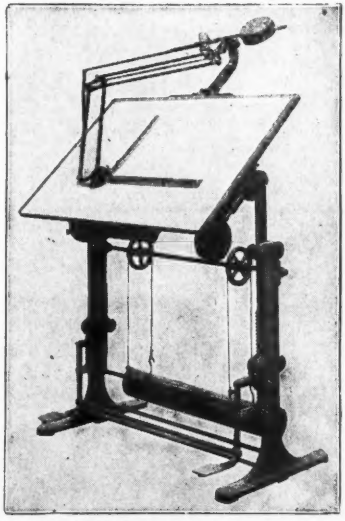
THE LIGHTEST, MOST EFFICIENT & COMPACT UNIVERSAL JOINT MANUFACTURED

Used on all types of Aircraft for engine controls, remote operation of various mechanisms, flap operating gear and undercarriages

MOLLART ENGINEERING COMPANY LIMITED
Precision and Production Engineers
KINGSTON-BY-PASS, SURREY

Designed and produced by proved Precision Engineers with Air Ministry Gauge Test House
Ref. No. 89755/31 Tel: ELMBRIDGE 3352/3/4/5

"MAVITTA" THE ALL-BRITISH DRAFTING MACHINES
(FOR ALL BOARDS)
ADJUSTABLE TABLES



THE MAVITTA DRAFTING MACHINES Ltd.
Anchor Works, Park Rd., Aston, Birmingham 6
Grams: Mavitta, Birmingham Phone: EAST 0482

INDEX TO ADVERTISERS

Aeroplane and Motor Aluminium Castings, Ltd. x
Airscrew Co., Ltd. xv
Armstrong Siddeley Motors, Ltd. ... vi
Armstrong Whitworth, Ltd. xi
Automatic Coil Winder & Electrical Equipment Co., Ltd. xx
Automotive Products, Ltd. vii
Avimo, Ltd. xiv

Bakelite, Ltd. ii
Birkett, T. M. & Sons, Ltd. viii
Birmetals, Ltd. ... ... —
Birmingham Aluminium Casting Co., Ltd. ... ... —
Blackburn Aircraft, Ltd. xx
Booth, James, & Company (1915), Ltd. xxiii
Bristol Aeroplane Co., Ltd. ... *Back Cover*
British Thomson-Houston Co., Ltd. ... vi
Bunhill Publications, Ltd. xvii
Burgess Products Co., Ltd. ... ... —
Butlers, Ltd. ... ... —

Cellactite & British Uralite Co., Ltd. —
Cementation Co., Ltd. ... ... *Back Cover*
Central Chemicals, Limited ... ... —
Cork Manufacturing Co., Ltd. ... xvi
Corrugated Packing & Sheet Metal Co., Ltd. ... ... —
Cunliffe-Owen Aircraft, Ltd. v

Dowty Equipment, Ltd. ix
Dunford & Elliott, Ltd. ... ... —
Dunlop Rubber Co., Ltd. iii

English Steel Corporation, Ltd. ... —

Fairey Aviation Co., Ltd. xix
Fescol, Ltd. ... ... —
Fielding & Platt Co., Ltd. ... ... —
Firth-Vickers Stainless Steels, Ltd. ... xviii
Firth, Thos. & Brown, John, Ltd. ... xvi
Fry's Metal Foundries, Ltd. ... ... —

General Aircraft, Ltd. xix

Habershon, J. J., & Sons, Ltd. ... ii
Handley Page, Ltd. ... ... —
Harcourt, David, Ltd. xiii
Hawker Aircraft, Ltd. ... ... —
High Duty Alloys, Ltd. xi
Hills, F. & Sons, Ltd. ... ... —
Hobson, H. M. (Aircraft & Motor) Components, Ltd. xiii
Hood, Haggie, R. & Son, Ltd. ... ... —
Hughes, F. A. & Co., Ltd. iv

Impervia, Ltd. xiv
Institute of Aeronautical Sciences ... xx
International Alloys, Ltd. ix
I.C.I. Plastics xii

Jablo Propellers, Ltd. xiv

Kolster-Brandes, Ltd. ... ... —
Klinger, Richard, Ltd. ... ... —

Lagonda Motors, Ltd. ... ... —
Lee, Arthur & Sons ... ... *Back Cover*
Leytonstone Jig and Tool Co., Ltd. xii
Lodge Plugs, Ltd. xv
London Name Plate Co., Ltd. ... xxi

Magnesium Castings & Products, Ltd. vii
Mavitta Drafting Machines, Ltd. ... xxi
Mollart Engineering Co., Ltd. ... xxi
Morrisons Eng. ... ... —
Multicore Solders, Ltd. viii

Napier, D. & Son, Ltd. ... ... —
National Savings Committee ... ... —
Newall, A. P. & Co. xxiii
Nitralloy, Ltd. ... ... —
Northern Aluminium Co., Ltd. ... —

Parnall Aircraft Co., Ltd. ... ... —
Peto & Radford viii
Pitmans ii

Reynolds Tube Co., Ltd. ... ... —
Roe, A. V. & Co., Ltd. x
Rolls-Royce, Ltd. v
Rotax, Ltd. iv
Rotol Airscrews, Ltd. ... ... —
Royal Air Force Quarterly ... ... —
Royal Aeronautical Society ... ... —

Sankey, J. & Sons, Ltd. ... ... —
Short Bros., Ltd. xviii
Short & Mason, Ltd. ... ... —
Simmonds Development Corporation, Ltd. xxii
Smith's Aircraft Instruments, Ltd. ... xiii

Technological Institute of Gt. Britain xxi

Vickers Armstrongs, Ltd. iii
Vokes, Ltd. ... ... —

Warwick Aviation Co., Ltd. ... ... —
Westland Aircraft, Ltd. ... ... —
Weston, Charles, & Co., Ltd. xii
Wilkinson Rubber Co. x

Yorkshire Engineering Supplies, Ltd. xvii

The SIMMONDS GROUP
LONDON
MELBOURNE
PARIS
NEW YORK
LOS ANGELES

INVENTORS

We continually seek new inventions and products in all branches of industry for development and production by the manufacturing companies of the Simmonds Group throughout the world.

SIMMONDS DEVELOPMENT CORPORATION LIMITED · BUSH HOUSE · WC2

Telephone: EAST 1221.

James Booth & Company (1915) Ltd.
Birmingham.

PIONEER MANUFACTURERS
IN GREAT BRITAIN OF
STRONG ALUMINIUM
AND
MAGNESIUM ALLOYS

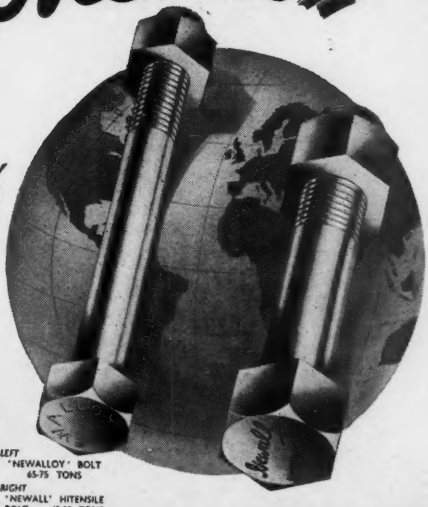
DURALUMIN
REGISTERED TRADE MARK

ALDURAL

MG

"ELEKTRON"

World Pioneers
IN THE MANUFACTURE of
Heat Treated
STEEL
BOLTS



LEFT 'NEWALLOY' BOLT 65-75 TONS
RIGHT 'NEWALL' HITENSILE BOLT 45-55 TONS

A.P.NEWALL & Co. Ltd. POSSILPARK GLASGOW·N

HIGH PERFORMANCE AIRCRAFT

HIGH POWER AERO ENGINES

BETONAC STEEL CONCRETE DUSTLESS FLOORS

AERO ENGINE FACTORY AND PRECISION MACHINE SHOP FLOORS REQUIRE TO BE DUSTLESS. DETAILS AND ESTIMATES SENT ON REQUEST.

THE CEMENTATION CO., LTD.
BENTLEY WORKS, DONCASTER & 39 VICTORIA ST., LONDON

AIRCRAFT STEELS

SPECIALISTS IN THE PRODUCTION OF—
Hot and Cold Rolled Steel Strip
Bright Drawn Steel Bars and Wire
Stainless Steel Bars, Strip and Wire

ARTHUR LEE & SONS, LTD.
SHEFFIELD, 9

London Office:
Mowbray House, NORFOLK ST., STRAND, LONDON, W.C.2
Tel. Nos.: Temple Bar 7187 & 7188

Birmingham Office:
155, CHURCH LANE, HANDSWORTH WOOD, BIRMINGHAM, 20
Tel. Nos.: Northern 2116 and 2117

Printed by the Lewes Press (Wightman & Co., Ltd.), Friars Walk, Lewes, England, and Published by the Royal Aeronautical Society, 4, Hamilton Place, London, W.1, England.

vy, 1943

AFT

ES

ORS
ST.

ON

D.

2117